THE APPLICATION OF DISCRETE WAVELET TRANSFORM FOR DIGITAL IMAGE COMPRESSION

Ahmad Khairul Umam[1)]
Pukky Tetralian Bantining Ngastiti[2)]
Aris Alfan[3)]
Zaqiyatus Shahadah[4)]
Amanda Fatma Muamalah[5)]
[1,2,3,4,5)] Program Studi Matematika, Fakultas Matematika dan Ilmu Pengetahuan Alam, Universitas Billfath, Lamongan Regency, Indonesia
e-mail: ahmad.khairul.umam@gmail.com

ABSTRACT

This article explains Discrete Wavelet Transform (DWT) in image compression. Wavelet transform is a generalization of Fourier transform, consisting of discrete and continuous wavelet transform. DWT has many uses, including image compression, fingerprint recognition, and image denoising. This research aims to know the steps of digital image compression using DWT and compare the original and resulting images. Efforts of DWT in digital image compression go by DWT's process, determining the threshold, sorting the absolute value of the image whether it is minor or more significant (equal to) threshold value, then is processed, Inverse Discrete Wavelet Transform (IDWT). This research explains the Peak Signal-to-Noise Ratio (PSNR), computing time, and compression ratio for three examples: the image of the cameraman, Lena, and a cat. The results determine that the highest PSNR values are wavelet of coiflets 3 for the cameraman, biorthogonal 3.5 for Lena, and coiflets 3 for the cat. The fastest computation times are wavelet of symlets 4 for the cameraman, symlets 4, coiflets 3 for Lena, and Daubechies 4 for the cat. Then, the highest compression ratios are wavelet of symlets 4, biorthogonal 3.5, coiflets 3 for the cameraman, Haar for Lena, and symlets 4, biorthogonal 3.5 for the cat. The results of this research are we get steps of the discrete wavelet transform for digital image compression. Also, we obtain types of wavelets with the highest PSNR values, the fastest computation times, and the highest compression ratios.

Keywords: discrete wavelet transform, image compression.

INTRODUCTION

Digital image compression is needed because the maximum size for sending a digital image is sometimes limited. Wavelet transform has many uses, for example, in digital signals. In digital signals, wavelet is used in the image compression process. According to Suma'inna & Alam (2014), wavelet is a data processing where signals selected are based on different frequencies. A wavelet function is divided into a scale function (father wavelet) and a wavelet function (mother wavelet). Based on Antoniou (2006), signals can be divided into continuous time signals and discrete-time signals, where discrete time signals are said to be digital signals. Based on Li et al. (2013), the basis of wavelet transform consists of Discrete Wavelet Transform (DWT) and complex number expansion of DWT itself, which is called Continuous Wavelet Transform (CWT). Wavelet from French is *ondelette* or a small wave. If translated into English, '*onde*' is changed to 'wave'. Then, combined with the original word, it is called 'wavelet'. There are types of wavelets, i.e., Haar wavelet, biorthogonal wavelet, symlets wavelet, Daubechies wavelet, etc. (Singh et al., 2014).

The image compression process is a topic in image processing. According to Budiansyah (2017), compression is a process of reducing the size of data so that you can get a compact digital image that can still represent the information in the data. Types of image compression are lossy compression and lossless compression. Compression based on transformation is lossy compression. So, image compression with DWT is lossy compression.

Image compression has lossless compression and lossy compression types. The explanation of lossy compression and lossless compression is as follows.

a. Lossless Compression

Based on Sunil Kumar & Indra Sena Reddy (2012), if data is compressed using a lossless compression method, the original data can be changed to the original image. This method is suitable for applying to data that needs to be similar to the original data.

b. Lossy Compression

Lossy compression removes some information from data, and then the data cannot be restored or reconstructed like the original image (Sunil Kumar & Indra Sena Reddy, 2012). This method is very suitable for objects such as videos, photos, et cetera (etc.).

There are many methods of image compression. The DWT method has been developed and has uniqueness and advantages compared to other methods. This research aims to evaluate image compression programs using DWT with Matlab. Besides that, it also compares the results of image compression using some wavelet wavelets for several images. Next, we compare the results of the Peak Signal-to-Noise Ratio (PSNR), computing times, and compression ratios.

Definition 1 (Andari, 2017) Let V is real vector space. Space V is called has inner product, if for all $\bar{u}, \bar{v}, \bar{w} \in V$ and all scalar k satisfy

a. $\langle \bar{u}, \bar{v} \rangle = \langle \bar{v}, \bar{u} \rangle$
b. $\langle \bar{u} + \bar{v}, \bar{w} \rangle = \langle \bar{u}, \bar{w} \rangle + \langle \bar{v} + \bar{w} \rangle$
c. $\langle k\bar{u}, \bar{v} \rangle = k\langle \bar{u}, \bar{v} \rangle$
d. $\langle \bar{v}, \bar{v} \rangle \geq 0$ dan $\langle \bar{v}, \bar{v} \rangle = 0 \Leftrightarrow \bar{v} = 0$

Real inner product space is real vector space together with the inner product.

Definition 2 (Depnath, 2002) Function space $L^2(\mathbb{R})$ is inner product space where that inner product is defined by

$$\langle f, g \rangle = \int_{-\infty}^{+\infty} f(x)\overline{g(x)}\, dx$$

with $f, g \in L^2(\mathbb{R})$.

Definition 3 (Bachman et al., 2012) Sequences in closed subsets $\{V_j, j \in \mathbb{Z}\}L^2(\mathbb{R})$ with function $\phi \in V_0$ is called multiresolution analysis if satisfy

a. ... $\subset V_{-1} \subset V_0 \subset V_1 \subset \cdots$(Increase).
b. $\bigcup_{J\in\mathbb{Z}} V_J = L^2(\mathbb{R})$(Dense).
c. $\bigcap_{J\in\mathbb{Z}} V_J = \{0\}$(Separable).
d. $f(t) \in V_j$ if and only if $f(2t) \in V_{j+1}$(Scale).
e. There is scale function $\phi \in V_0$ with translation to integers $\{\phi(t-n): n \in \mathbb{Z}\}$ which is an orthonormal basis on V_0(Orthonormal Basis).

Definition 4 (Depnath, 2002) Wavelet is a class of functions that are built from translation and dilation, which is defined by

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right), a, b \in \mathbb{R}, a \neq 0$$

where a is the dilation/scale parameter, and b is the translation parameter. A function $\psi(t)$ in a class of functions is the mother wavelet.

Wavelet can be constructed by mother wavelet $\psi(t)$ and father wavelet (scale function) $\phi(t)$. A wavelet constructed by a scale function is $\phi_{a,b}(t) = 2^{\frac{a}{2}}\phi(2^a t - b) \in L^2(\mathbb{R})$. Then, a wavelet constructed by the mother wavelet is $\psi_{a,b}(t) = 2^{\frac{a}{2}}\psi(2^a t - b) \in L^2(\mathbb{R})$. DWT is image decomposition by starting with the image row decomposition process and continuing with the image column decomposition process. (Suma'inna & Alam, 2014)

For example, the original image $f(x, y)$ and denoised image $\hat{f}(x, y)$ where both images are of size $M \times N$. The Root Mean Square Error (RMSE) value is

$$RMSE = \sqrt{\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left(\hat{f}(x,y) - f(x,y)\right)^2}.$$

According to Yin (2012), the PSNR value is

$$PSNR = 20\log_{10}\left(\frac{255}{RMSE}\right).$$

Al-Shereefi (2013) discusses compression only using Daubechies wavelets for rontgen images, satellite images, and fingerprints. In their paper, Suma'inna & Alam (2014) compared image compression with two types of wavelets: Haar and Daubechies wavelets. In the paper, Singh et al. (2014) compared image compression with three wavelets: Daubechies, Haar, biorthogonal, and symlets wavelets. This paper compares image compression results using Haar, symlets, Daubechies, coiflets, and biorthogonal wavelets for grayscale images of a cameraman, Lena, and a cat. We also compare the results of PSNR, computing time, and compression ratio.

METHOD

Concept and Literature Study

We study concepts and look for references about image compression, DWT, and image compression using DWT.

Object of Research

We study image compression, DWT, and image compression using DWT. Then, we create the Kc-TWD algorithm for grayscale image compression by selecting images from the literature. The selected image has a size of 512×512 pixels.

Steps of Research

Steps of research:

a. We construct a discrete wavelet transform. We study the properties of DWT and image compression.

b. We create an image compression algorithm using DWT. Then, we implement that algorithm using Matlab.

Numerical Simulation

Our test images are grayscale: a cameraman, Lena, and a cat with a size of 512×512 pixels. We use the DWT method for digital image compression using Matlab R2015a on the Windows 7 operating system with an Intel Atom CPU 1,86 GHz processor and 32-bit graphics. We show the original image and the image after the compression process.

RESULTS AND DISCUSSION

DWT on a signal $f(t)$:

$$TWD\{f(t)\} = W_{\phi(a_0,b)} + W_{\psi(a,b)} \tag{1}$$

with

$$W_{\phi(a_0,b)} = \frac{1}{\sqrt{M}} \sum_{t=0}^{M-1} f(t)\phi_{a_0,b}(t)$$

$$W_{\psi(a,b)} = \frac{1}{\sqrt{M}} \sum_{t=0}^{M-1} f(t)\psi_{a,b}(t)$$

for $t = 0, 1, 2, 3, \dots, M-1$ and $a \geq a_0$. IDWT from (1):

$$f(t) = \frac{1}{\sqrt{M}} \sum_{b=0}^{2^a-1} W_{\phi(a_0,b)}\, \phi_{a_0,b}(t) + \frac{1}{\sqrt{M}} \sum_{a=a_0}^{\infty} \sum_{b=0}^{2^a-1} W_{\psi(a,b)} \psi_{a,b}(t)$$

with $a_0 = 0, a = 0, 1, 2, \dots, A-1$, and $M = 2^A$. Decomposition (DWT) and reconstruction (IDWT):

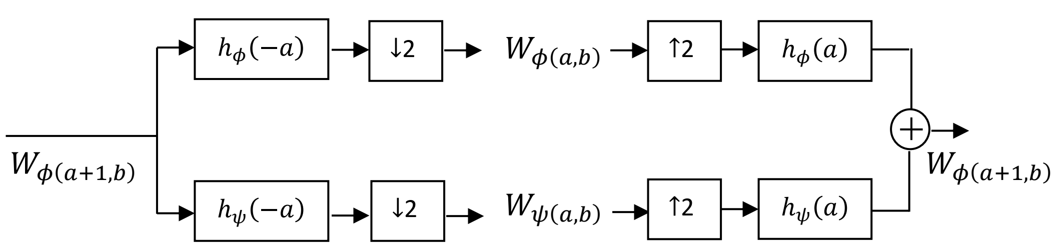

Figure 1. DWT and IDWT 1-D

with $h_\phi(a), h_\psi(a), \uparrow 2, \downarrow 2, \oplus$ are scale function coefficient (lowpass filter), wavelet function coefficient (highpass filter), upsampling, downsampling, and direct sum.

DWT on image $C(m,n)$ with size $M \times N$:

$$TWD\{C(m,n)\} = W_{\phi(a_0,b_1,b_2)} + W^j_{\psi(a,b_1,b_2)} \qquad (2)$$

with

$$W_{\phi(a_0,b_1,b_2)} = \frac{1}{\sqrt{M \times N}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} C(m,n)\, \phi_{a_0,b_1,b_2}(m,n)$$

$$W^j_{\psi(a,b_1,b_2)} = \frac{1}{\sqrt{M \times N}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} C(m,n)\, \psi^j_{a,b_1,b_2}(m,n)$$

where $j = h, v, d$ and $h =$ horizontal, $v =$ vertical, $d =$ diagonal. IDWT from (2):

$$C(m,n) = \frac{1}{\sqrt{M \times N}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} W_{\phi(a_0,b_1,b_2)} \phi_{a_0,b_1,b_2}(m,n)$$

$$+ \frac{1}{\sqrt{M \times N}} \sum_{j=h,v,d} \sum_{a=a_0}^{\infty} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} W^j_{\psi(a,b_1,b_2)}\, \psi^j_{a,b_1,b_2}(m,n)$$

with $a_0 = 0, a \geq a_0, a = 0, 1, 2, \ldots, A-1$, and $M = N = 2^A$.

Decomposition (DWT) for digital image

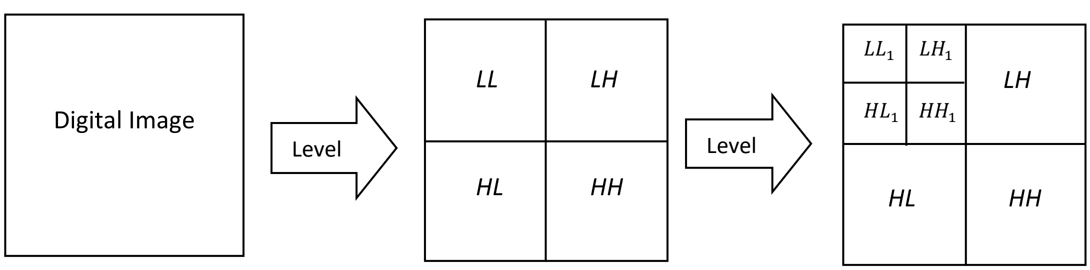

Figure 2. DWT for Digital Image Levels 1 and 2

where image decomposition level 1 are *LL, LH, HL,* and *HH* (average coefficient and horizontal, vertical, and diagonal detail coefficients). Wavelet decomposition can be continued up to levels 3, 4, 5, etc., by using an average image at each level.

The Flowchart of image compression using DWT is presented in Figure 3. Steps of image compression using DWT:

a. Image decomposition using DWT.

b. Determine the value of the threshold.
c. If an image's absolute value is greater than the threshold, then the value of the image is fixed.
d. If the absolute value of an image is less than or equal to the threshold, then the image value is 0.
e. Image reconstruction using IDWT.

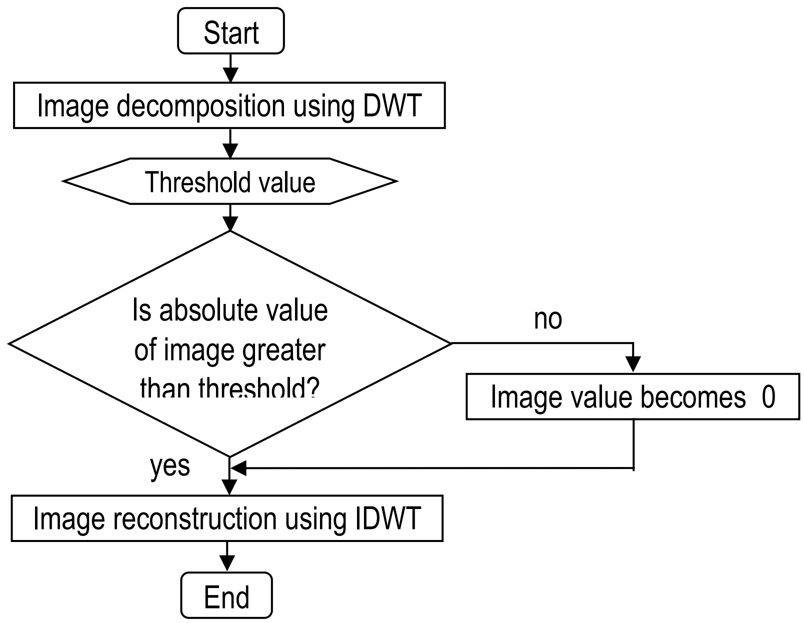

Figure 3. Flowchart of Wavelet Transform Algorithm for Digital Image Compression

Images of a cameraman, Lena, and a cat before and after are being compressed using a Haar wavelet with a threshold value of 25 (Figures 4, 5, and 6). The size of the original and after compression of the cameraman's image are 40 KB and 32 KB, respectively. The PSNR value for the cameraman's image is 28.28. In comparison, the size of the original and after-compression images of Lena are 524 KB and 218 KB, respectively. The PSNR value for Lena's image is 28.65. The size of the original image, image after compression, and PSNR value for the cat image are 62 KB, 36 KB, and 28.35.

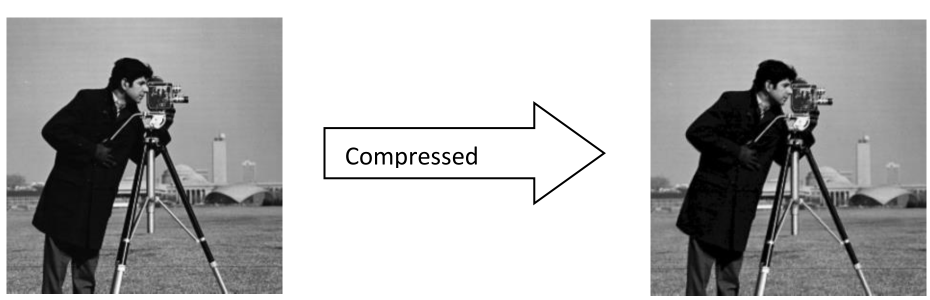

Figure 4. Cameraman Image Before and After Compressing with Wavelet Haar

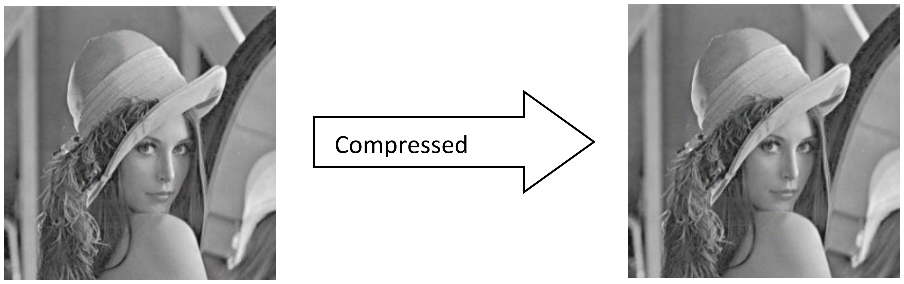

Figure 5. Lena Image Before and After Compressing with Wavelet Haar

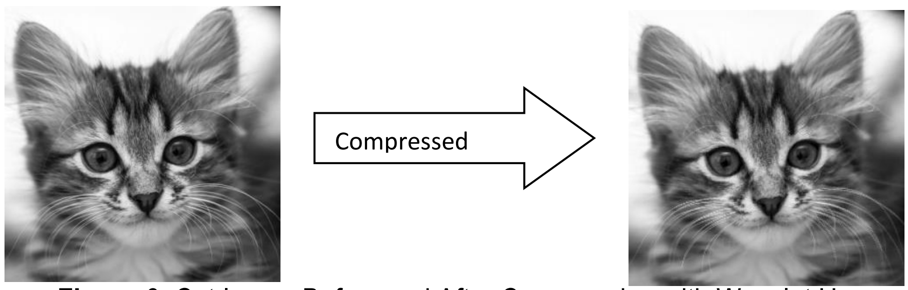

Figure 6. Cat Image Before and After Compressing with Wavelet Haar

We do 35 experiments with a threshold value of 25. The highest PSNR values are for coiflets 3, biorthogonal 3.5, and coiflets 3 wavelets (see Table 1). The fastest computing time are symlets 4 wavelet for the cameraman, symlets 4 and coiflets 3 wavelets for Lena, and Daubechies 4 wavelet for the cat (see Table 2). The most significant compression ratios are symlets 4, biorthogonal 3.5 and coiflets three wavelets for the cameraman, Haar wavelet for Lena, and symlets 4 and biorthogonal 3.5 wavelets for the cat (see Table 3).

Table 1. Average Value of PSNR

	Haar	**Symlets 4**	**Daubechies 4**	**Coiflets 3**	**Biorthogonal 3.5**
Cameraman	28,28	34,66	33,85	35,27	35,12
Lena	28,65	30,86	30,91	31,09	31,12
Cat	28,35	30,14	30,33	30,34	30,09

Table 2. Average of Computing Times (seconds)

	Haar	**Symlets 4**	**Daubechies 4**	**Coiflets 3**	**Biorthogonal 3.5**
Cameraman	1,23	1,2	1,24	1,24	1,27
Lena	4,57	4,56	4,89	4,56	4,59
Cat	2,07	2,02	1,99	2,21	2,07

Table 3. Average of Compression Ratio (%)

	Haar	**Symlets 4**	**Daubechies 4**	**Coiflets 3**	**Biorthogonal 3.5**
Cameraman	20	22,5	20	22,5	22,5
Lena	58,4	16,03	18,51	16,22	19,27
Cat	42,86	44,44	42,86	42,86	44,44

Discrete wavelet transform can also be applied for digital image denoising. There is also the Dual-Tree Complex Wavelet Transform (DTCWT), an improvement on the discrete wavelet transform where the imaginary part is the Hilbert transform. Besides that, a wavelet transform can be developed into a quaternion wavelet transform. A quaternion wavelet transform has a basis like the Fourier quaternion transform, but this is only specific to translation and dilation.

CONCLUSION

In this research, we succeeded in analyzing discrete wavelet transform theory, and we continued its application for digital image compression. We did 35 experiments and got wavelet types with the highest PSNR values, fastest computing time, and most significant compression ratio for grayscale images of the cameraman, Lena, and cat with the size of 512×512 pixels.

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