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Problem-Solving Framework in Algebra Through Backward Error-Analysis

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A B S T R A C T

This study aimed to propose Backward Error-Analysis (BEA) as an alternative assessment method in algebra by evaluating its relevance and effectiveness and developing a problem-solving framework for classroom use. Specifically, it compared the Conventional Assessment Method and BEA in evaluating learners' algebra performance, tested BEA's effectiveness, assessed learners' perceptions including general perceptions and challenges and developed a BEA-based problem-solving framework. A Quantitative Research Design was used involving 60 Grade 9 learners from Pili National High School, divided into two groups: one using conventional assessment and the other BEA. Both groups took pre-tests and post-tests, with two interventions conducted for the BEA group, and a self-constructed survey assessed learners' perceptions. Findings showed no significant difference between the two groups' test results, yet BEA proved effective as learners' post-test scores significantly improved. Learner perceptions highlighted factors such as engagement, perceived usefulness, efficiency, learning outcomes, and commitment, with challenges including emotional resistance and cognitive struggles in error analysis. The study also introduced the "BEA-Trick Framework" as a foundation for classroom implementation. Overall, BEA was found effective and offered unique advantages over conventional methods. It is recommended for classroom adoption to foster 21st-century skills and higher-order thinking.

INTRODUCTION

Algebra is a fundamental branch of mathematics that serves as a gateway to higher mathematical learning and problem-solving skills. This involves mathematical operations and symbols as well as the application of various rules for manipulating these symbols. Like every other field in mathematics, learners often encounter struggles in solving algebra problems. Learners sometimes get wrong answers due to errors in their procedure and misconceptions, while others face the opposite, getting the right answers but with erroneous parts in the solution. Mulungye et al. (2016) highlighted that these errors in algebra are brought about by misconceptions that are at times shared and thus spread among the learners. These errors can be factual, procedural, or conceptual, which identifies their possible root causes (Cheng, 2012).

Studies suggested that exposing learners to incorrect solutions and having them correct mistakes can be a powerful learning strategy. Booth et al. (2013) found that learners who analyzed incorrect algebraic solutions developed a better understanding of algebraic principles compared to those who only practiced correct solutions. By examining errors, learners engage in a deeper cognitive process that strengthens their comprehension of mathematical concepts. Likewise, Siegler (2002) argued that learning from errors leads to deeper conceptual retention, as it forces learners to actively engage in cognitive conflict and resolution. This process encourages learners to reflect on their thinking, evaluate their misconceptions, and ultimately develop a more robust understanding of mathematical principles.

Constructivist theories emphasize that learning is most effective when learners actively participate in the knowledge construction process (Piaget, 1952; Vygotsky, 1978). In the context of mathematics education, Hiebert et al. (1996) asserted that learners learn best when they engage in tasks requiring them to explain their reasoning. This perspective aligns with the principles of active learning, where learners develop deeper understanding through interaction, exploration, and critical thinking. Engaging learners in meaningful discussions, problem-solving activities, and reflective thinking allows them to make connections between new concepts and prior knowledge. When learners are encouraged to articulate their reasoning, justify their solutions, and analyze their mistakes, they gain a stronger grasp of mathematical ideas. Additionally, social interaction, as emphasized by Vygotsky, plays a crucial role in learning, as peer discussions and teacher-guided instruction facilitate knowledge construction.

Various methods aligned with these studies and theories are utilized in educational schemes, with error analysis being the most notable. It is regarded as an effective tool for identifying areas of learning that require development. Some approaches also encourage learner engagement by having students assess their own responses, identify errors, and correct them through reflective reasoning. However, despite its recognized benefits, most studies have examined error analysis primarily as a diagnostic or instructional tool, rather than as a structured assessment strategy. Few have explored its potential as an alternative method for evaluating learners' performance and understanding of mathematical concepts and problem-solving. Moreover, conventional error analysis typically follows a forward approach, in which learners solve problems, receive feedback on errors, and attempt to correct them. While effective, this method often focuses on surface-level corrections rather than addressing underlying misconceptions and promoting deeper mathematical reasoning.

This study proposes to introduce and assess the effectiveness of the reverse process of error analysis, coined in this study as Backward Error Analysis (BEA), as an alternative assessment tool in algebra. BEA shifts the responsibility to learners by requiring them to analyze algebraic problems that contain intentional errors. Their task is to locate these errors in the given solutions, correct them, and provide justifications for the mistakes within an assessment framework. This assessment strategy has the potential to not only provide educators with an engaging method for evaluating learners’ algebraic proficiency but also to deepen learners’ conceptual understanding, enhance their problem-solving skills, and improve knowledge retention.

METHODOLOGY

This chapter presents the overall framework and methodology used by the researcher in conducting the study. It includes research methods, research design, respondents of the study, data gathering procedures, and statistical tools.

Research Design

This quantitative research utilized a descriptive correlational approach to evaluate the algebra performance of learners using both the Conventional Method of Assessment and Backward Error Analysis (BEA) and to make a comparison between the two. The purpose of the assessment was to find out if there was a significant difference between the two methods and to evaluate the effectiveness of BEA as a tool for assessing the outcome. The study further intended to accumulate empirical evidence in support of the possible classroom application of BEA. Learners from two different groups participated in the research and took pre-tests and post-tests in algebra, each test consisting of 10 questions.
The first group was assessed by the conventional method, and the second group by BEA. Performance between the two groups was compared through independent sample tests while the pre-test and post-test results were analyzed within the BEA group using dependent sample tests. In the BEA assessments, learners had to point out, correct, and give reasoning for the intentional errors that were incorporated into the test items.

A descriptive approach was adopted to assess the students' perceptions of BEA, including their overall impressions and difficulties experienced during its execution. Data was collected via a questionnaire survey using Likert-scale items. The results from this part were used for identifying the need for improvements and for providing the recommendations to make BEA a better alternative assessment strategy in algebra.

Research Method

The research used a quantitative method to investigate the correlation between a Conventional Assessment Method and Backward Error Analysis (BEA) and to assess the latter's effectiveness as a method of assessing learners' performance in algebra. A correlational design was applied to the two methods of assessment by comparing the results obtained from pre-tests and post-tests, using tests of difference between independent samples for group comparisons and dependent samples for evaluating learning gains within the BEA group. The evaluation of learners' views on BEA was done with the help of a descriptive method via a Likert-scale survey questionnaire, and the data were analyzed descriptively to provide guidance for the improvement and development of BEA as a strategy for assessing students' knowledge of algebra.

Respondents of the Study

A group of ninth graders from Pili National High School was involved in the study. They were selected as the group to be tested because they are at the very beginning stage of learning algebra and therefore, erroneous ideas are prevalent among them. The study consisted of two 30 learner groups, one evaluated through the Conventional Method, and the other through Backward Error Analysis (BEA). The two groups did pre-tests and post-tests, respectively, and the comparison of their results was done to assess the effectiveness of BEA as a substitution assessment technique in algebra.

Data Gathering Procedure

The research created a 10-item pre-test and post-test for each participating group where the Conventional Method and Backward Error Analysis (BEA) were used to record learners' algebra performance. Besides, the learners were given 5-item BEA tests in the intervening period between the pre-test and post-test to get them accustomed to the BEA method. Moreover, a home-made survey questionnaire was created to gather learners' views on BEA, which was subsequently validated for content by subject experts and subjected to reliability testing through a pilot study conducted with Grade 9 learners from different schools.

Once approvals were granted, the pilot study was performed, and then data collection was initiated at Pili National High School. The participants were assigned to the Conventional Method Group and BEA Group through purposive matching based on their previous semester algebra scores, to ensure baseline equivalence between the two groups.

Statistical Tool

Research employed quantitative approaches to analyze students' ability to solve algebraic problems during two assessment methods: the Conventional Method and Backward Error Analysis (BEA). The study aimed to find the effect of BEA on learners' performance by comparing pre-test and post-test scores of students who were taught using BEA. Also, differences in learners' performance of the Conventional Method and BEA were analyzed to find out if there was a considerable superiority of one method over the other. A self-structured scoring system was employed to evaluate students' BEA answers, and each item was scored on a scale of 1 to 5 based on the quality of the response, with the help of previously defined descriptors (see Table 1).

Table 1. BEA Assessment Tool Scoring Rubric

Score	Descriptor
5	All errors were correctly identified and fixed with clear and accurate justifications of the mathematical concepts applied.
4	All errors were identified and fixed, but justifications were unclear, incomplete, or contained minor inaccuracies.
3	All errors were identified, but only some were fixed, with little to no justification or mostly incorrect reasoning.
2	Only a few errors (less than half) were identified, with minimal or no corrections and justifications.
1	No errors were correctly identified or addressed.

RESULT AND DISCUSSION

This chapter presents the analysis and interpretation of data gathered from Grade 9 learners of Pili National High School, Pawili, Pili, Camarines Sur. The study evaluated the effectiveness of Backward Error-Analysis (BEA) in assessing learners' algebra performance and examined learners' perceptions of BEA as an alternative assessment tool. Data was analyzed using appropriate statistical techniques and guided by established research standards. The discussion of results is organized according to the specific research objectives of the study.

Comparison of the Conventional Assessment Method and the Backward Error- Analysis (BEA) in Assessing Learners' Algebra Performance

This study conducted a comparison between the proposed Backward Error-Analysis (BEA) and the Conventional Assessment Method in terms of their effectiveness in assessing learners' algebra performance. The researchers utilized parametric analysis for this comparison after ensuring that assumptions for the test were met (Shapiro-Wilk p-value=0.433, Levene's test p-value=0.057, at 5% significance level, with random sample, and no significant outliers detected).

As a result, an independent sample T-test was conducted to compare the mean scores of the two groups. The test yielded a non-significant result (p = 0.922, at the 5% significance level), as shown in Table 2. This indicates that there was no statistically significant difference in the assessment results between learners who were assessed using the Traditional Method and those assessed through Backward Error Analysis.

Table 2. Mann-Whitney U Test Between BEA and Conventional Assessment

U	df	p
POT	354.500	0.922

Note. For all tests, the alternative hypothesis specifies that group *BEA* is greater than group *Conventional.*

Legend:
U – Mann-Whitney test statistic
POT – post-test
df – degrees of freedom
p – p-value

Since learners performed similarly under both BEA and traditional assessments, this finding suggests that Backward Error Analysis (BEA) can be effectively adopted and implemented in the classroom alongside traditional assessment methods without negatively affecting academic outcomes. Moreover, BEA serves as a valuable complementary tool for assessing algebra performance given its notable advantages such as fostering conceptual understanding and enhancing long-term retention of mathematical concepts–learning benefits that are not consistently emphasized in traditional assessment methods.

Table 3. Advantages of Conventional Assessment Method and Backward Error-Analysis

Conventional Assessment Method	Backward Error-Analysis (BEA)
Reinforces memory and foundational knowledge through repeated exposure and practice using drills, quizzes, and standardized tests.	Promotes better retention by engaging learners in analyzing, correcting, and understanding intentional errors during the problem-solving process.
Focuses on procedural fluency and accuracy by encouraging the use of standard algorithms and step-by-step solutions.	Fosters deeper conceptual understanding by requiring learners to identify, correct, and explain the mistakes in the problem.
Encourages structured thinking by following clear solution paths that reinforce systematic approaches.	Enhances critical and analytical thinking through justification of errors identified and corrections made.

Table 3 presents a comparison between the traditional assessments and Backward Error-Analysis (BEA). Traditional methods mainly concern recall and the correctness in the procedure, and they are easy to perform but they often highlight memorization and do not recognize the higher-order skills. BEA, on the other hand, has students doing error analysis, which initiates and nourishes the process of deeper understanding of concepts, critical thinking, metacognition, and reflective learning. There was no significant difference between student performance in the two methods, but BEA added educational value without lowering academic outcomes. This is in line with earlier studies and learning theories that regard mistakes as a necessary component of learning process. In short, BEA is a powerful ally of traditional assessments, especially in mathematics where the emphasis is on understanding rather than merely following correct procedures.

Effectiveness of Backward Error-Analysis (BEA) as an Assessment Tool

This study examined the effectiveness of Backward Error-Analysis (BEA) in improving learners' algebraic problem-solving skills. Using dependent samples t-test, pre-test and post-test results were compared after confirming that all parametric assumptions were met. The analysis yielded a statistically significant result (p = 0.007), indicating a significant improvement in post-test scores. These findings provide strong evidence that BEA is an effective teaching and assessment tool for enhancing algebraic problem-solving performance (see Table 4).

Table 4. Wilcoxon Signed-Rank Test of BEA Pre-Test and Post-Test Results

Measure 1	Measure 2	W	z	df	p
Post-test	Pre-test	331.500	2.465		0.007

Legend:
W – Wilcoxon signed-rank statistic
df – degrees of freedom
z – score literal
p – p-value

The enhanced post-test scores reveal that BEA effectively supported learners' error identification and misconception recognition, as well as their corrections done through reflective thinking. The activities grounded in BEA involved the learners in the full cycle of errors analysis, applying corrective techniques, and reasoning through the solutions, thus it reinforced their comprehension of math concepts. These outcomes signify that BEA is not only an evaluation instrument but also a teaching approach that encourages the development of higher-order thinking skills, problem-solving and reflective learning, which are the dimensions that traditional assessments often overlook.

These results are consistent with Singh (2023), who pointed out that error analysis cultivates a growth mindset by considering errors as opportunities for learning. In the same manner, McLaren et al. (2015) proved that the practice of analyzing and rectifying wrong examples enhances one's understanding of concepts and the ability to solve problems. Along with the impressive performance on the tests, these studies advocate for Backward Error Analysis (BEA) as a powerful method for teaching and assessing that not only makes mathematical understanding stronger but also prepares learners with the necessary 21st-century skills.

Perceptions of Learners on Backward Error-Analysis (BEA) as a Teaching and Assessment Approach

This study also explored how learners perceive BEA in terms of both their general attitudes and the challenges they encounter during its application, particularly in solving algebraic problems. The data were divided into two main themes: *(1) General Perception of Learners* and *(2) Challenges Faced by Learners*, both situated within the problem-solving framework in algebra through Backward Error Analysis. Each theme was analyzed using factor analysis with oblimin rotation, which enabled the identification of latent constructs underlying the learners' responses.

Learners' General Perceptions of BEA

This section presents the analysis and interpretation of the factor loadings related to learners' general perceptions within the Problem-Solving Framework in Algebra through Backward Error-Analysis (BEA). Factor analysis was conducted to categorize general perceptions into meaningful components. As a result, three primary factors emerged, representing key dimensions that shaped learners' views and experiences with BEA. These factors are discussed in detail in Table 5.

Table 5. Factor Loadings on General Perceptions

	Factor 1	Factor 2	Factor 3	Uniqueness
GPL14	0.741			0.361
GPL6	0.721			0.447
GPL21	0.694	0.410		0.247
GPL5	0.645			0.552
GPL9	0.601	0.448		0.379
GPL1	0.598			0.630
GPL19	0.553			0.574
GPL8	0.498			0.586

	Factor 1	Factor 2	Factor 3	Uniqueness
GPL12	0.456			0.649
GPL3	0.442			0.783
GPL16	0.422			0.739
GPL18		0.785		0.359
GPL17		0.733		0.426
GPL7		0.676		0.526
GPL22		0.645		0.472
GPL20		0.404	0.406	0.621
GPL2			0.657	0.578
GPL11			0.639	0.570
GPL13			0.585	0.589
GPL23			0.558	0.662
GPL10			0.485	0.486
GPL4				0.737
GPL15				0.867
GPL24				0.823
GPL25				0.841

Note. The applied rotation method is oblimin.

Factor 1: Engagement & Attitudes Toward BEA

Factor 1 reflects learners' positive perceptions of Backward Error Analysis (BEA) as an effective instructional and learning tool. The results indicate that learners view BEA as constructive in enhancing understanding, correcting misconceptions, and promoting active learning through reflective error analysis. Although some items (see Figure 1) show higher uniqueness suggesting the influence of factors such as teaching style or prior experiences they still load meaningfully on this factor, indicating a robust and consistent perception of BEA's positive contribution to learning outcomes. High loadings for items such as GPL14 and GPL6 demonstrate strong consensus among learners that BEA fosters critical thinking, supports learning from mistakes, and enhances engagement, reinforcing its potential value in curriculum design and classroom practice.

According to Yusof and Selamat (2017) and Van der Meijden and De Lange (2020), learners become more engaged when introduced to structured, interactive, and reflective analytical tools similar to Backward Error Analysis. Likewise, García-Sánchez and Moreno-Guerrero (2016), and Schunk and DiBenedetto (2020) emphasized that strategies centered on error reflection enhance learners' interest, motivation, emotional engagement, and confidence. These findings align with the results of Factor 1, further supporting the integration of BEA as an engaging and effective instructional approach that promotes deeper conceptual understanding and reflective learning in mathematics education.

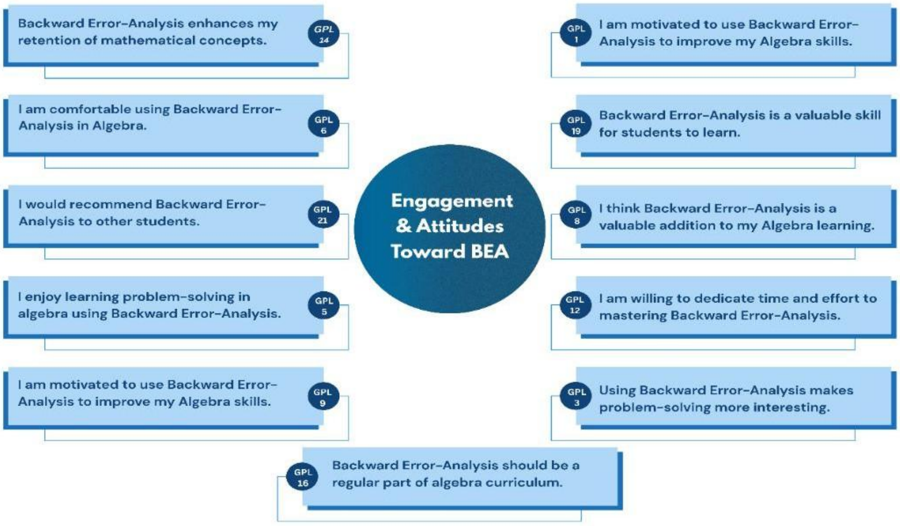

Figure 1. Engagement & Attitudes Toward BEA

Factor 2: Perceived Usefulness and Efficiency

These items (see Figure 2) indicate that Factor 2 reflects learners' engagement with Backward Error Analysis (BEA) and its function as a diagnostic feedback tool. BEA encourages active involvement in the learning process by helping learners identify misunderstandings, reflect on errors, and understand correct reasoning, thereby enhancing metacognition. The presence of item cross-loadings with Factor 1 suggests that learners perceive BEA's instructional value and engagement as closely interconnected, highlighting its multifaceted role as both a teaching strategy and a self-assessment mechanism. This overlap reinforces the validity of Factor 2 in capturing the motivational and diagnostic qualities of BEA.

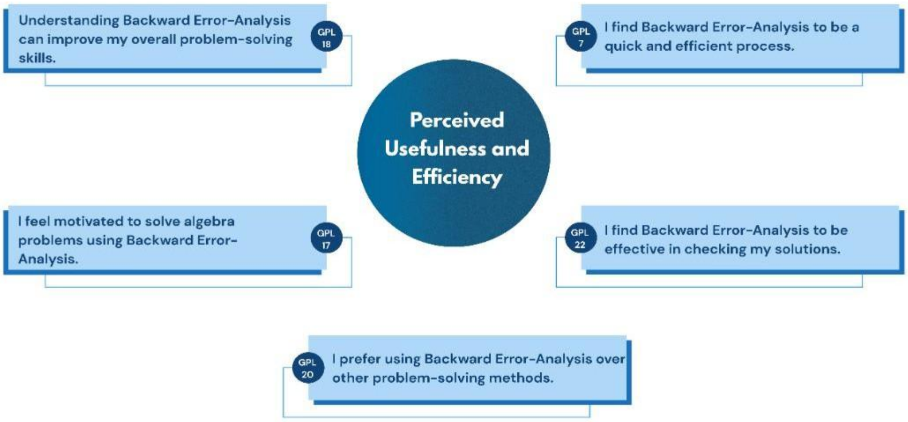

Figure 2. Perceived Usefulness and Efficiency

Factor 2 reveals that learners are not passive recipients of feedback but active participants in the learning process. The strong loadings indicate a positive reception of BEA's capacity to help identify misconceptions and reinforce correct reasoning. As such, educators can utilize BEA not only as a summative assessment method but also as a formative tool that keeps learners mentally engaged and diagnostically informed about their progress and areas for improvement.

Abuzaid and Al-Dosari (2018) found that structured strategies enhance learners' perceptions of usefulness, aligning with learners' views of BEA as valuable for checking solutions and exam preparation. Similarly, Sezer and Ozan (2017) reported that error analysis is appreciated for its efficiency and time-saving qualities, while Lazarus and Rosenthal (2021) emphasized its role in identifying logical errors and improving clarity and confidence. Together, these studies reinforce BEA's effectiveness as a diagnostic and learner-centered assessment approach.

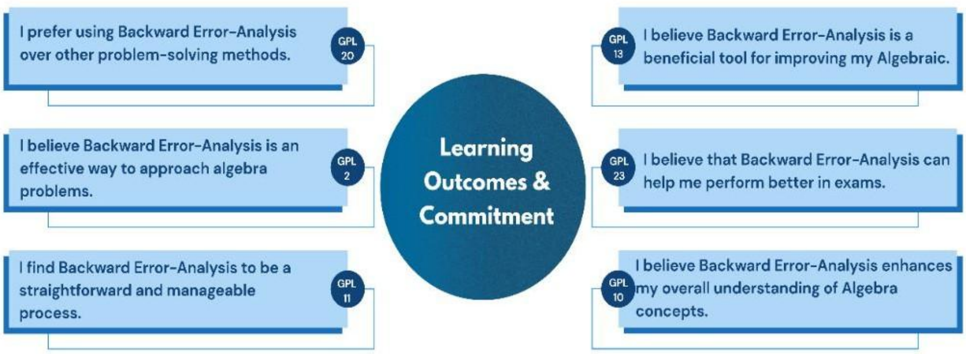

Figure 3. Learning Outcomes & Commitment

Factor 3 reflects learners' mixed and uncertain perceptions of Backward Error Analysis (BEA). While some learners recognize its benefits, higher uniqueness values and weaker loadings suggest uncertainty related to its complexity, cognitive demands, or difficulty in interpreting error-focused tasks. Several items that do not load strongly on any factor further indicate that individual experiences, contextual influences, or misunderstandings of BEA shape these perceptions. Overall, this factor highlights that although BEA is generally viewed positively, a subgroup of learners may experience hesitation or confusion, underscoring the need for clearer instruction, guided practice, and scaffolding when implementing BEA.

Pintrich (2016) emphasized that learners' belief in the effectiveness of a learning method influences their commitment and performance, which aligns with learners' willingness to invest effort in BEA for its perceived benefits in algebra problem-solving. Similarly, Ali and Khan (2019) and Bergman and Högberg (2020) found that reflective, error-based strategies enhance learning outcomes, retention, and conceptual understanding. Zimmerman and Schunk (2017) further noted that goal-oriented and reflective approaches strengthen self-regulation and academic habits. Collectively, these studies support the findings of Factor 3, suggesting that with appropriate support and clarity, learners' initial uncertainty toward BEA can be transformed into sustained engagement and improved learning outcomes.

The findings from the factor analysis suggest that Backward Error Analysis (BEA) is perceived by learners as an engaging, effective, and efficient tool that not only enhances their understanding of algebra but also positively influences their learning outcomes and commitment. The three identified factors; Engagement and Attitudes Toward BEA, Perceived Usefulness and Efficiency, and Learning Outcomes and Commitment, provide a comprehensive perspective on how learners experience and benefit from BEA. These insights contribute to a deeper understanding of how BEA can be strategically integrated into algebra instruction to foster motivation, academic success, and strong problem-solving skills.

Additionally, some items exhibited high uniqueness values and did not load significantly on any of the identified factors. These items may reflect learner perceptions that are not closely aligned with the dominant themes captured in the analysis or may indicate areas where responses were more variable or uncertain. Further qualitative investigation may be warranted to better understand these outliers and uncover nuanced perspectives that quantitative methods alone may not fully reveal.

Challenges Faced by Learners with BEA

This section presents the analysis and interpretation of the factor loadings related to the challenges encountered by learners within the Problem-Solving Framework in Algebra through Backward Error Analysis (BEA). Factor analysis was employed to categorize these challenges into meaningful components, resulting in the emergence of two primary factors that represent distinct types of difficulties experienced by learners. These factors are discussed in Table 6.

Factor 1: Emotional and Motivational Resistance to Error Analysis

This factor (see Figure 4) reflects learners' emotional and motivational resistance to engaging in Backward Error Analysis (BEA). Learners reported feelings of frustration, discouragement, and confusion when tasked with identifying and correcting errors, particularly those they did not make themselves. This emotional discomfort can lead to disengagement and a perception that error correction is futile, reducing the likelihood of meaningful learning in algebra. The findings highlight that a significant portion of learners experience challenges not only cognitively but also emotionally when participating in BEA tasks.

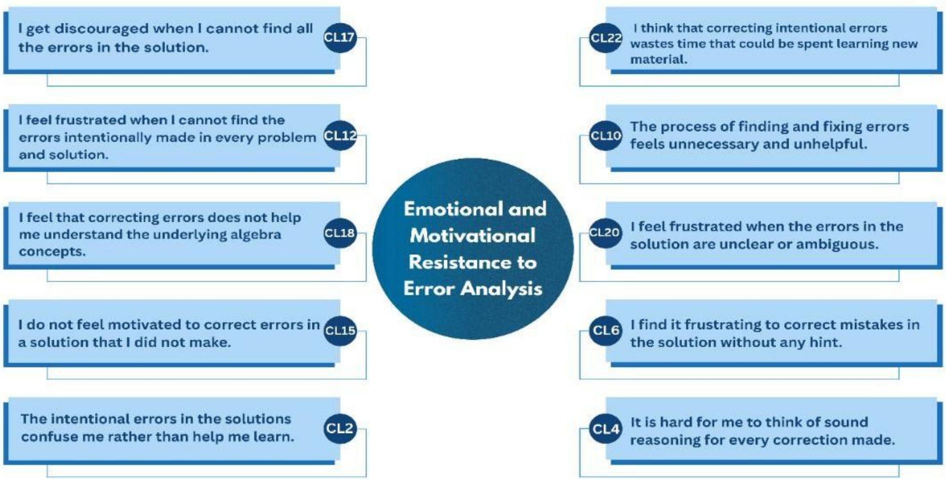

Figure 4. Emotional and Motivational Resistance to Error Analysis

Research emphasizes the critical role of emotions in learning from errors. Borasi (1996) noted that errors serve as effective learning tools only when learners are emotionally prepared to engage with them. Ashcraft and Krause (2007) demonstrated that math anxiety impairs working memory, making reflective error analysis more difficult, especially in abstract areas like algebra. Tulis (2013) found that negative emotions such as embarrassment reduce engagement, while Boaler (2016) argued that traditional punitive approaches to mistakes foster fixed mindsets. Collectively, these studies suggest that without emotional support, scaffolding, and clear pedagogical framing, BEA may be perceived as frustrating or unproductive. Addressing learners' emotional responses is therefore essential to maximize the effectiveness of BEA in promoting meaningful learning.

Table 6. Factor Loadings on Challenges Met

	Factor 1	Factor 2	Uniqueness
CL17	0.789		0.406
CL12	0.695		0.486
CL18	0.686		0.516
CL15	0.684		0.491
CL2	0.657		0.539
CL22	0.618		0.612
CL10	0.613		0.464
CL20	0.536		0.701
CL6	0.513		0.693
CL7	0.421	0.483	0.473
CL9	0.418		0.837
CL4	0.404		0.658
CL14		0.752	0.449
CL24		0.745	0.490
CL13		0.689	0.468
CL23		0.606	0.611
CL25		0.559	0.631
CL1		0.500	0.649
CL3			0.886
CL5			0.941
CL8			0.904
CL11			0.589
CL16			0.828
CL19			0.691
CL21			0.930

Note. Applied rotation method is oblimin.

Factor 2: Cognitive Struggle with Error Identification

The second factor reflects learners' cognitive challenges in engaging with Backward Error Analysis (BEA), highlighting perceptions that identifying and understanding intentional errors adds complexity and detracts from meaningful learning (see Figure 57). Learners reported difficulty focusing on their own understanding while analyzing incorrect solutions, questioning the educational value of BEA. Unlike emotional resistance, this factor represents cognitive resistance, where learners struggle to reconcile the cognitive demands of error analysis with perceived learning benefits, sometimes viewing the process as more hindrance than help.

Cognitive Load Theory (Sweller, 1988) explains that tasks exceeding working memory capacity can hinder learning, which is evident in BEA's dual demand of analyzing errors while understanding algebraic principles. VanLehn (1999) and Durkin and Rittle-Johnson (2012) similarly noted that error-based strategies require strong metacognitive skills, and learners often fail to spontaneously recognize their instructional value. Chi et al. (1989) emphasized that learners must perceive tasks as meaningful to benefit fully; otherwise, engagement is superficial and learning transfer is limited. Collectively, these studies support the finding that cognitive resistance arises from perceived misalignment between the effort required and the educational value, underscoring the need for scaffolding and clear guidance to help learners engage effectively with BEA.

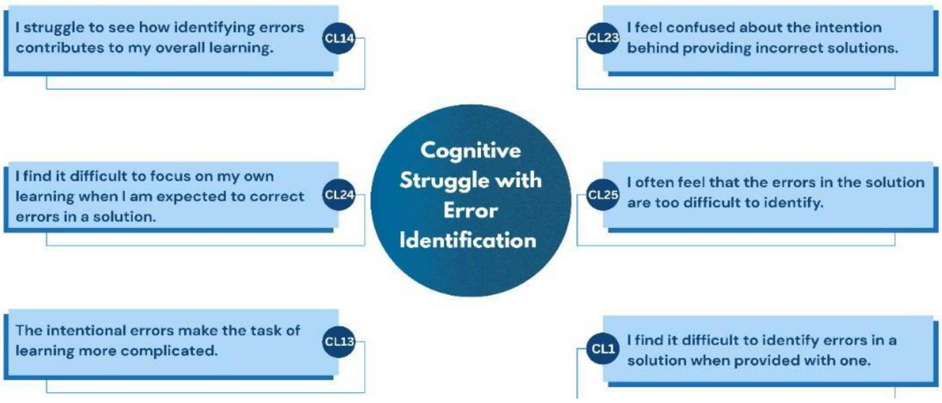

Figure 5. Cognitive Struggle with Error Identification

Problem-Solving Framework in Algebra Through Backward Error Analysis

This study primarily aimed to propose Backward Error Analysis (BEA) as an alternative assessment method in algebra and to evaluate its effectiveness for potential classroom implementation. A series of BEA-based testing interventions were conducted to assess its capacity to measure learners' algebraic performance and to compare its relevance and advantages over traditional assessment methods. Additionally, learner perceptions of the BEA tests were collected and analyzed to identify key factors that may contribute to enhancing and refining this form of assessment.

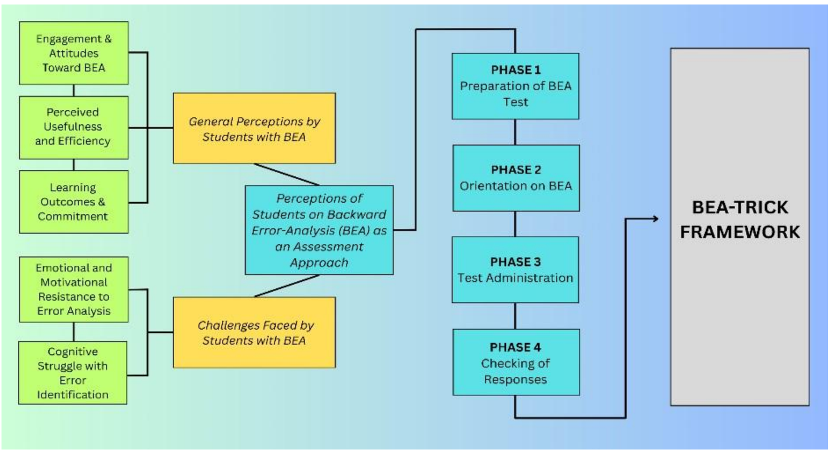

Figure 6. Problem-Solving Framework in Algebra Through Backward Error-Analysis

The study revealed that Backward Error Analysis (BEA) effectively identifies how learners approach algebra problems and highlights common misconceptions, focusing on reasoning and problem-solving steps rather than solely on correct answers. This process encouraged students to critically reflect on their work, fostering deeper understanding and conceptual retention. Teachers also gained valuable insights into learners' thinking, enabling targeted interventions. While BEA produced results comparable to traditional assessments, the significant improvement in post-test scores demonstrates its effectiveness as both an assessment tool and a strategy for enhancing learners' comprehension and long-term retention of mathematical concepts.

Based on the results of this investigation, a problem-solving framework in algebra using BEA was developed and named the "BEA-Trick Framework" (see Figure 6). This framework outlines how BEA can be effectively implemented as an assessment method in mathematics, providing clear guidelines and procedures for its application. It also highlights essential considerations for educators when adopting this approach. As such, the BEA-Trick Framework offers a foundational model for the potential integration of BEA into regular classroom assessment practices, either as a complementary tool or as an alternative to conventional methods.

The framework follows a linear progression, moving from left to right, and culminating in the output of this research. Positioned at the left-most section are the key factors identified as essential for the further development and successful implementation of Backward Error-Analysis (BEA) in the classroom setting. These factors were derived from the learners' perceptions, which were systematically examined in this study. Specifically, the framework incorporates insights into learners' general attitudes toward BEA as an assessment tool, as well as the challenges they encountered during its application in algebra problem-solving.

As illustrated, the left-most section of the framework emphasizes the key factors influencing the potential implementation of Backward Error-Analysis (BEA), which are organized into two main categories: (1) learners' general perceptions of BEA, and (2) the challenges they encountered during its application. These factors are visually linked to the learners' perceptions, which were systematically evaluated in this study. From this point, a directional flow guides the framework toward the phases, steps, or guidelines for implementing BEA, signifying that any future application should be informed by these identified perceptions and challenges to promote an effective, learner-centered approach.

Ultimately, the progression culminates in the BEA-Trick Framework, which represents the intended outcome of this study. This final stage underscores that when learners' experiences and feedback are thoughtfully integrated into the planning and implementation of BEA, the assessment method effectively fulfills its objective—enhancing mathematical understanding through reflective, error-based learning.

Factors on General Perceptions

Engagement & Attitudes Toward BEA

This reflects learners' positive perceptions of BEA as an effective tool for learning and assessment. High loadings indicate strong agreement that it enhances understanding, critical thinking, and motivation in Algebra. BEA promotes active learning, emotional engagement, and interest in problem-solving, making mathematics more enjoyable and meaningful.

Perceived Usefulness and Efficiency

This highlights learners' view of BEA as a practical and motivational tool. It helps identify misconceptions, improve reasoning, and enhance metacognition, while supporting accuracy, time efficiency, and exam preparation. BEA's dual role as a formative and summative tool increases confidence and academic performance.

Learning Outcomes & Commitment

This captures learners' mixed perceptions of BEA. While many recognize its benefits for problem-solving and deeper learning, some feel challenged by its complexity. Research shows that reflective strategies like BEA improve understanding, retention, and study habits, emphasizing the need for clear guidance to foster engagement and commitment.

Factors on Challenges Met

Emotional and Motivational Resistance to Error Analysis

This factor reflects learners' emotional reluctance to engage in BEA. Many feel frustrated, discouraged, or confused when correcting unclear or intentional errors, perceiving the task as pointless. Research shows that math anxiety, negative emotions, and fixed mindsets contribute to disengagement, highlighting the need for supportive approaches that frame errors as learning opportunities.

Cognitive Struggle with Error Identification

This factor captures learners' difficulties in recognizing and interpreting intentional errors. Cognitive resistance arises from the dual demands of understanding incorrect solutions while applying underlying principles, often leading to overload. Without proper scaffolding, learners perceive error analysis as distracting, overwhelming, and unproductive, limiting meaningful learning and metacognitive growth.

The following factors that should be considered were the procedures or guidelines on how BEA can be implemented. These procedures, composed of four phases, detail the step-by-step process that must be completed to make BEA not just effective but also beneficial, especially for the learners.

Guidelines/Procedures on Implementing Backward Error-Analysis as an Assessment Tool in Algebra

PHASE 1: Preparation of BEA Test

The preparation of BEA test items requires thoughtful planning and attention to detail. Teachers must first construct algebraic problems complete with step-by-step solutions. These worked-out solutions serve as the basis for introducing intentional, minimal errors either within the steps or in the final answer. The task of the learners is to detect and correct these errors, providing justifications for their corrections. However, it is essential to maintain a balance in the number of errors per item; exceeding three errors per problem is discouraged, as this may cause confusion and increase difficulty for the learners. Likewise, including too many problems in one test is not advisable, given the cognitive demand and the time required to process and solve each BEA item effectively, as observed in the testing phases of this study.

PHASE 2: Orientation on BEA

Before administering the BEA assessment, it is crucial to conduct a comprehensive orientation for the learners. The orientation should explain the purpose and process of BEA, from error identification to justification, and clearly outline the time limits and expectations for the test. Moreover, learners must understand the significance of BEA in enhancing their mathematical understanding and reasoning skills. Presenting BEA not as a more difficult form of assessment, but as an opportunity for meaningful learning and error-based reflection, can foster a more positive learner attitude and increase their engagement with the method.

PHASE 3: Test Administration

After the orientation, the BEA test can be administered to the learners. Each learner should receive the test materials and be given a specific amount of time to complete the tasks. This phase replicates standard testing conditions, with the added cognitive demand of analyzing, identifying, and justifying corrections to the provided algebraic solutions.

PHASE 4: Checking Responses

The evaluation of learner responses should be guided by a scoring rubric developed specifically for the BEA framework. This rubric, introduced in the study, assigns a maximum of five points and a minimum of one point per item. The scoring must focus not only on whether the learners corrected the errors accurately, but also on the depth and clarity of their justifications.

Once the procedures are completed and the identified factors have been considered, Backward Error-Analysis (BEA) is successfully achieved. Following the established guidelines/procedures and necessary considerations ensures the effective implementation of this assessment method. This rationale explains why the "BEA-Trick Framework" was positioned at the rightmost part of the model, symbolizing the goal the successful accomplishment of Backward Error-Analysis.

CONCLUSION

The lack of significant differences in learners' performance indicated that BEA could be used alongside traditional assessments without compromising educational outcomes. This suggested that BEA was a valid alternative assessment method in algebra education. While BEA may not have led to higher test scores compared to conventional methods, it offered valuable pedagogical benefits, such as fostering deeper understanding, critical thinking, and analytical reasoning. This aligned with educational theories that emphasized the importance of learning from mistakes. Additionally, BEA aligned with constructivist and metacognitive approaches to learning, which prioritized the learning process over mere correctness. This supported the idea that engaging with errors could enhance learners' conceptual understanding and metacognitive skills.

Backward Error-Analysis (BEA) was an effective tool for assessing and enhancing learners' algebraic problem-solving skills. The significant increase in post-test scores demonstrated that BEA not only aided in evaluating learner performance but also served as a powerful teaching method that fostered critical thinking and a growth mindset. By encouraging learners to reflect on their mistakes and understand the reasons behind them, BEA transformed errors into valuable learning opportunities, ultimately leading to improved mathematical understanding and problem-solving abilities.

Learners perceived Backward Error-Analysis (BEA) as an effective and engaging assessment tool in algebra, contributing positively to their learning experiences. The findings suggested that BEA fostered critical thinking, enhanced metacognition, and encouraged active participation in the learning process. However, the presence of uncertainty among some learners highlighted the need for clearer guidance and support in implementing BEA. The unique perceptions captured in certain items also indicated that individual experiences with BEA varied significantly, suggesting that a one-size-fits-all approach might not have been sufficient.

The BEA-Trick Framework represents a significant advancement in leveraging Backward Error-Analysis for educational assessment. By detailing implementation factors and procedural guidelines, it equips educators with a practical tool for fostering algebraic problem-solving skills. This framework holds promises for enhancing both formative and summative assessments, thereby contributing to improved teaching strategies and student learning outcomes. Its demonstrated effectiveness and adaptability affirm its value in modern educational practices, encouraging further exploration and application across diverse learning contexts.

Limitations And Recommendations

This study is subject to several limitations that should be acknowledged. First, the sample was limited to 60 Grade 9 learners from a single school (Pili National High School, Philippines), which restricts the generalizability of findings to other educational contexts, school types, or grade levels. Second, the sample size per group (n = 30) is insufficient for the exploratory factor analyses conducted on the 25-item perception questionnaires, as factor analysis generally requires a minimum of 5 to 10 respondents per item; future studies should aim for a larger sample (n >= 125). Third, only two BEA intervention sessions were conducted, which may be insufficient to detect the full long-term impact of BEA on conceptual retention. Fourth, the study relied exclusively on quantitative instruments; complementary qualitative components such as think-aloud protocols or post-assessment interviews would provide richer insights into learners' metacognitive processes during BEA tasks.

For future research, it is recommended that: (1) BEA be replicated with larger and more diverse samples across multiple schools and grade levels to establish broader external validity; (2) longitudinal studies be conducted to examine BEA's long-term effects on algebra achievement and knowledge retention; (3) a mixed-methods design incorporating qualitative data collection be employed to triangulate and deepen quantitative findings; and (4) the BEA-Trick Framework be subjected to design-based research for iterative refinement in authentic classroom settings.

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