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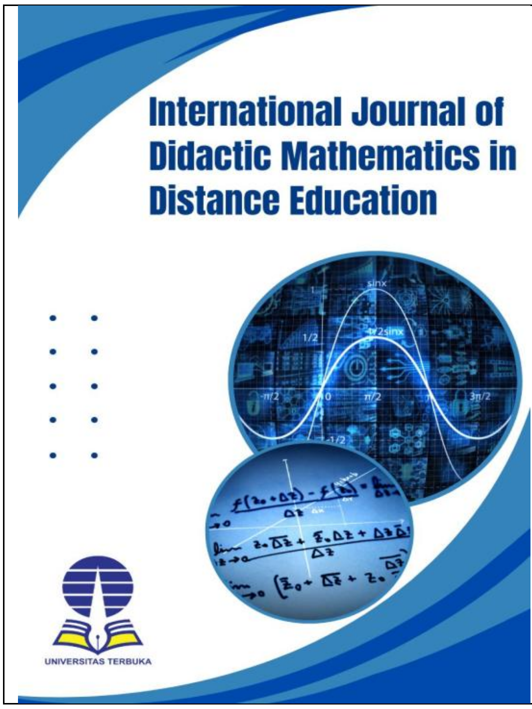


# Exploring neuro-mathematical connections in the resolution of a contextualized geometric problem

**Benilda María Cantillo-Rudas**
Faculty of Education, Universidad de Panamá, Panamá, benilda-m.cantillo-r@up.ac.pa

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# Exploring neuro-mathematical connections in the resolution of a contextualized geometric problem

**Benilda María Cantillo-Rudas**
Faculty of Education, Universidad de Panamá, Panamá, benilda-m.cantillo-r@up.ac.pa

**Abstract**
This study aims to explore the neuro-mathematical connections activated by high school and university students when solving a contextualized geometric problem. The urgency of this research lies in the need to deepen our understanding of the cognitive and neurological processes involved in mathematical problem-solving, particularly in spatial reasoning tasks. The theoretical framework combines Connections Theory and the Onto-semiotic Approach, focusing on the typology of neuro-mathematical connections. The qualitative, descriptive methodology was carried out in three phases: (1) selection of volunteer participants from high school and university levels; (2) data collection through the application of a geometric problem involving the volume of two boxes, with video recordings capturing students' problem-solving processes; and (3) analysis using the theoretical framework to identify and interpret the neuro-mathematical connections activated during the task. The results revealed a rich network of cognitive processes encompassing mathematical practices, objects, processes, and semiotic functions. Specifically, students demonstrated: recognition of mathematical terms and symbols; activation of visual perception, spatial reasoning, and motor coordination; association of concepts and formulas; execution of intermediate calculations and unit conversions; sequential problem-solving; and reflective verification of results. These findings support the claim of the Extended Theory of Connections that connections are inherently cognitive processes. This research contributes to the field of mathematics education and cognitive science by providing an in-depth analysis of how students engage with mathematical problems through neuro-mathematical pathways. Future research should expand this work by incorporating neuroimaging or eye-tracking technologies to further validate and visualize the cognitive mechanisms underlying mathematical reasoning.

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## 1. Introduction

In Mathematics Education, multiple curricular organizations emphasize the importance of students and teachers establishing connections between mathematical concepts to strengthen their understanding (MEN, 2006; Rodríguez-Nieto et al., 2024). Research has shown that one of the main difficulties in learning mathematics lies in the lack of these relationships, which impacts problem-solving and the construction of meaningful knowledge (Mhlolo, 2012; Rodríguez-Nieto et al., 2025). Furthermore, in various fields of knowledge, the ability to establish connections between mathematical ideas is fundamental for deep understanding and the development of new perspectives. In science, the constant dialogue between theories drives revolutionary discoveries and paradigm shifts. Likewise, in literature and history, the deep interplay between contexts and events enhances the interpretation of facts, offering a more profound understanding of human thought and societal evolution (Alsina, 2020).

Most studies focus on algebra, exploring how teachers make connections with the quadratic equation (Businskas, 2008), calculus and understanding of exponential and logarithmic functions (Campo-Meneses & García-García, 2023), and the reversibility between the derivative and the integral in pre-university students (García-García & Dolores-Flores, 2018, 2021). University students' understanding of the derivative has also been analyzed from the perspective of network theories (Rodríguez-Nieto et al., 2021a, 2021b, 2021c, 2022a), as well as the relationship between the rate of change and the conceptualization of slope (Dolores-Flores et al., 2019). Currently, connection theory has been used to analyze the mathematical activity of university students when solving multivariate calculus problems (Rodríguez-Nieto & Font, 2025a).

Other research has addressed the connections that future teachers make in geometric problems (Caviedes-Barrera et al., 2023) and teachers' understanding of the concept of function (Hatisaru, 2022). Furthermore, connections have been studied in classroom contexts, where the interaction between teachers and students favors the relationship between concepts, procedures, and demonstrations. It is suggested that evidence of extra-mathematical connections be included in future research (De Gamboa et al., 2023). In higher education, studies have explored how students make connections when solving arithmetic progression problems, highlighting patterns and representations in their reasoning processes (Bortoli & Bisognin, 2023). From a sociocultural perspective, the relationship between ethnomathematics and STEAM education has been examined, applying them to everyday practices such as the preparation of Mexican tacos (Rodríguez-Nieto et al., 2022b; Rosa & Orey, 2021). However, there are still no studies that analyze what happens in the mind before making mathematical connections.

Neuroscience has investigated the construction of geometric solids based on APOS theory (Giraldo-Rojas et al., 2021) and has analyzed the law of trichotomy from a neuroscientific perspective (Osler, 2012; Osler & Mason, 2016). Neuroimaging studies have shown that the brain processes mathematical information abstractly, with the left hemisphere playing a key role in recognizing oral and written language, numeracy, and mathematical logic (Serna, 2020). The brain gathers information through the senses, processes it, and generates responses following an attentional process. The posterior-superior parietal system is involved in mathematical attention, supporting spatial orientation and the mental representation of quantities (De la Serna, 2020). Additionally, the parietal cortex contributes to understanding written language and solving mathematical problems, playing a fundamental role in attention, numerical processing, and working memory. Alterations in this region can lead to dyscalculia, a condition characterized by difficulties in numerical comprehension (Cantillo-Rudas & Rodríguez-Nieto, 2024; Cantillo-Rudas et al., 2024; Rousselle & Noel, 2007; Rubinsten & Henik, 2005).

The comprehension and verbal expression of numbers occur in the language area of the dominant hemisphere, usually the left. The symbolic representation of numbers is processed in the middle ventral occipitotemporal cortex and the fusiform gyrus, while the abstract representation of magnitudes involves the intraparietal sulci of both hemispheres, establishing a distributed neural network for mathematical comprehension and calculation (Serna, 2020). The triple code model (Dehaene & Cohen, 1995; Dehaene et al., 2003) proposes three mental systems: (1) the quantity system or "number sense," for comparing and estimating magnitudes; (2) the verbal system, responsible for memorizing and recognizing arithmetic facts; and (3) the visual system, which represents numbers in different symbolic formats, facilitating comparison and calculation. These processes depend on the superior parietal lobe, which is essential for numerical cognition and arithmetic learning. When numerical meaning and the physical representation of digits do not match, this leads to interference in mathematical tasks, affecting reaction times (Girelli et al., 2000; Henik & Tzelgov, 1982; Rubinsten et al., 2002). The early ability to count and estimate quantities is linked to the development of advanced mathematical skills (Libertus et al., 2013).

The literature review emphasizes the importance of integrating neuromathematics to promote meaningful learning. However, challenges persist in teaching geometric figures, as students struggle to define and calculate the volume of various prisms. Additionally, they find it difficult to identify prisms and establish connections between two-dimensional figures and their three-dimensional counterparts. Given these challenges, understanding the neuro-mathematical connections involved in learning geometry is essential, as they play a crucial role in conceptual development and problem-solving (Cantillo-Rudas et

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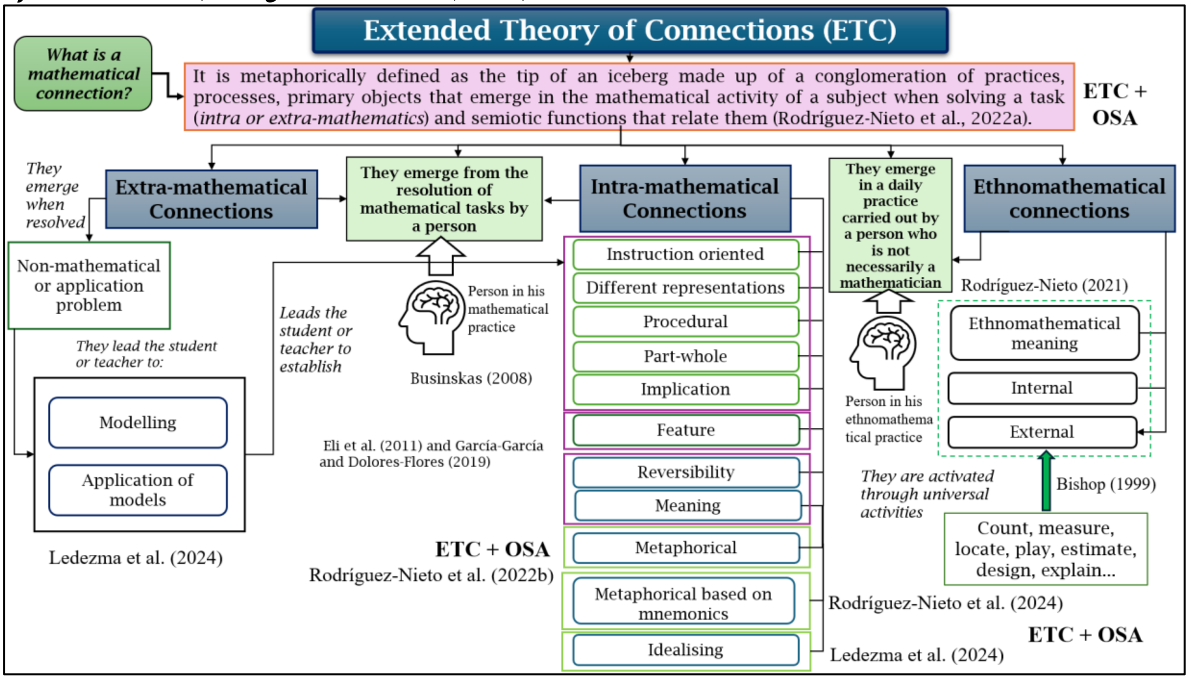
al., 2024; Downton & Livy, 2022; Ruiz-Soto, 2018; Vásquez, 2019). Consequently, this study aims to analyze the neuro-mathematical connections activated by high school and university students, as well as a high school teacher, when solving a contextualized geometric problem.

## 2. Theoretical basis

### 2.1 Extended Theory of Connections in Mathematics Education

In this theory, a mathematical connection is understood from view of the ETC and the OSA as the tip of an iceberg made up of a conglomerate of practices, processes/objects (problem situations, languages, procedures, propositions, definitions, and arguments), and semiotic functions (SFs) that relate them (Rodríguez-Nieto et al., 2022a). In ETC two groups of connections are identified: the intra-mathematical, extra-mathematical connections and ethnomathematical connections (Dolores-Flores & García-García, 2017; Rodríguez-Nieto et al., 2024). In this work, we only consider the intra-mathematical connections (ver Figura 1) .

Figure 1
*Synthesis of ETC (Rodríguez-Nieto et al., 2024)*




1) Modelling: refers to the relationship that a person establishes between the world of mathematics and the real world (or the daily life of students) and between mathematics and other sciences. It can be understood as the connection formed between a mathematical concept and a real-world task (either occurring or potentially occurring in everyday life) or a practical application in a field outside of mathematics. In this process, the subject constructs a mathematical model based on the task to find a solution. When the subject builds the mathematical model, he uses various knowledge (mathematical or not) by executing multiple actions (algebraic, symbolic, graphic, etc.) to reach an answer consistent with the requirement posed (Campo-Meneses & García-García, 2023; Dolores-Flores & García-García, 2017; Evitts, 2004).
2) Instruction-oriented: It refers to the understanding and application of a mathematical concept D derived from two or more related concepts, B and C. These connection types can be recognized in two forms: 1) the relationship of a new topic with previous knowledge, and 2) the mathematical concepts, representations, and procedures connected are considered fundamental prerequisites that people must have to develop new content (Businskas, 2008). For example, when the teacher explains to the students that, in order to work on the partial derivative of a function, they must first recall the concepts of functions, limits, the global derivative, and the slope of a line.

3) Procedural: these connections are identified when a student uses rules, algorithms, or formulas to solve a mathematical problem. They are of the form, A is a procedure to work with a concept B (García-García & Dolores-Flores, 2021).
4) Different representations: They are identified when the subject represents mathematical objects using equivalent (same register) or alternate representations (different registers) (Businskas, 2008).
5) Feature: These connections are identified when the student expresses some characteristics of the concepts or describes their properties in terms of other concepts that make them different from or similar to the others (Eli et al., 2011).
6) Reversibility: They occur when a student starts from concept A to obtain a concept B and inverts the process, starting from concept B to return to concept A (Adu-Gyamfi et al., 2017; García-García & Dolores-Flores, 2021).
7) Part-whole: They occur when logical relationships are established in two ways. The first refers to the generalization relation of form A and is a generalization of B and B is a particular case of A. The second is that the inclusion relationship is given when a mathematical concept is contained within another (Businskas, 2008).
8) Meaning: this mathematical connection is identified when a student attributes a meaning to a mathematical concept or use it in solving a problem (García-García & Dolores-Flores, 2020).
9) Implication: these connections are identified when a concept A leads to another concept B through a logical relationship (Businskas, 2008; Selinski et al., 2014).
10) Metaphorical: these connections are understood as the projection of properties, characteristics, etc. of a known domain to structure another lesser-known domain (Rodríguez-Nieto et al., 2022b).
11) Metaphorical connections based on mnemonics: this connection is "understood as the relationship established by the subject between a mnemonic rule (often a familiar resource) and a mathematical object, rule, or mathematical procedure to memorize and use strategically more easily" (Rodríguez-Nieto et al., 2024, p. 18). These types of connections are both inclusive and recursive, with three key elements to consider: a) keywords that are similar to the word (or term) being referenced; b) acronyms, which are formed when the first letter of each word in a list is used to construct a new word. c) acrostics which consist of constructing a sentence, where the first letter of each constitutes the term studied (Mastropieri & Scruggs, 1989; Rodríguez-Nieto et al., 2024).
12) Idealising: this connection relates an ostensive to a non-ostensive. Its function is to dematerialize the ostensive and turn it into an ideal mathematical object (for example, the bottom of a tank rounded is considered circle/circumference) (Ledezma et al., 2024).

This theory of mathematical connections can continue to be extended and the work of Cantillo-Rudas et al. (2024) stands out, where they report new developments on neuro-mathematical connections associated with the cognitive part of this theory and the assessment of the brain areas activated in a person's mathematical activity.

### 2.2. Onto-semiotic Approach (OSA)

The OSA emerged in response to the need to clarify and organize various fundamental notions that allow us to describe and understand the cognitive phenomena involved in the learning and teaching of mathematics. Since the early 1990s, within the framework of a theoretical course on Mathematics Education in the doctoral program at the University of Granada, Spain, the urgent need to clarify concepts such as knowledge, conception, concept, schema, operational invariant, meaning, and praxeology was identified. Thus, the OSA has established itself as a key tool for the in-depth and articulate analysis of mathematical activity, both in the learning and teaching processes, by offering a theoretical framework that integrates the cognitive, epistemic, and semiotic dimensions of said activity (Godino et al., 2024).

Furthermore, OSA considers that to describe mathematical activity from an institutional and personal point of view, it is essential to have in mind the objects involved in such activities and the semiotic relations between them (Font et al., 2013). Mathematical activity is modeled in terms of practices, the configuration of primary objects, and processes that are activated by practices. Mathematical practice is considered in this theory as a sequence of actions, regulated by institutionally established rules, guided toward a goal (usually solving a problem). In the OSA ontology, the term 'object' is used in a broad sense to

*International Journal of Didactic Mathematics in Distance Education*
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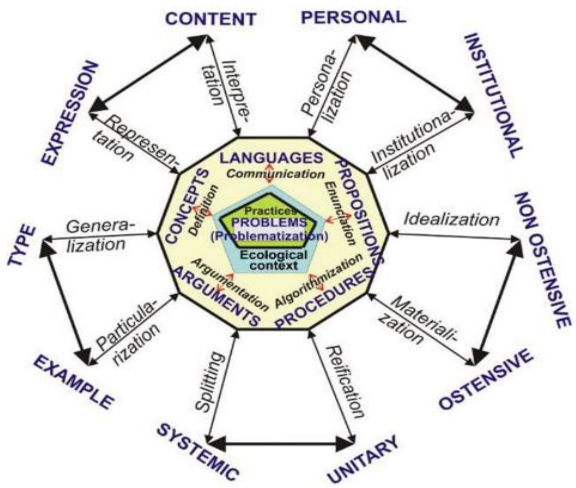
https://jurnal.ut.ac.id/index.php/ijdmde

refer to any entity which is, in some way, involved in mathematical practice and can be identified as a unit. For example, when carrying out and evaluating a problem-solving practice, we can identify the use of different languages (verbal, graphic, symbolic, …). These languages are the ostensive part of a series of definitions, propositions, and procedures that are involved in the argumentation and justification of the solution of the problem. Problems, languages, definitions, propositions, procedures, and arguments are considered objects, specifically as the six mathematical primary objects. Taken together they form configurations of primary objects. The term configuration is used to designate a heterogeneous set or system of objects that are related to each other. Any configuration of objects can be seen both from a personal and an institutional perspective, which leads to the distinction between cognitive (personal) and epistemic (institutional) configurations of primary objects. The OSA also considers processes, understood as a sequence of practices involving configurations of primary objects.

The mathematical objects that intervene in the mathematical practices and those that emerge from them may be considered from the perspective of the following ways of being/existing, which are grouped into facets or dual dimensions (Font & Contreras, 2008; Font et al., 2013): extensive-intensive (intensive objects correspond to those collections or sets of entities, of whatever nature, which are produced either extensively, by enumerating the elements when this is possible, or intensively, by formulating the rule or property that characterizes the membership of a class or type of objects), expression-content (the objects may be participating as representations or as represented objects), personal-institutional (institutional objects, emerge from systems of practices shared within an institution, while personal objects emerge from specific practices from a person), ostensive-non ostensive (something that can be shown directly to another person, versus something that cannot itself be shown directly and must therefore be complemented by another something that can be shown directly), and unitary-systemic (the objects may participate in the mathematical practices as unitary objects or as a system).

Problem solving is achieved through the articulation of sequences of practices. Such sequences take place over time and are often considered processes. In particular, the use and/or the emergence of the primary objects of the configuration (problems, languages, definitions, propositions, procedures, and arguments) takes place through the respective mathematical processes of communication, problematization, definition, enunciation, elaboration of procedures (algorithmization, routinization, etc.), and argumentation (applying the process-product duality). Meanwhile, the dualities described above give rise to the following processes: institutionalization – personalization, generalization – particularization, analysis/decomposition – synthesis/reification, materialization/concretion – idealization/abstraction, expression/representation – meaning (Font et al., 2013). In Figure 2 onto-semiotic configuration of practices, objects and processes.

Figure 2
*Onto-semiotic configuration of practices, objects and processes (Godino, 2014, p. 23)*




This list of processes derived from the typology of primary objects and dual facets used as tools to analyze mathematical activity in OSA, while contemplating some of the processes considered as important

International Journal of Didactic Mathematics in Distance Education
Volume 2, Issue 1, pp. 58-80, E-ISSN: 3047-9207
https://jurnal.ut.ac.id/index.php/ijdmde

in mathematical activity, is not intended to include all the processes involved in that activity. This is because, among other reasons, some of the most important processes, such as problem solving and mathematical modeling, are a macro processes (as a set of processes) rather than just mere processes (Godino et al., 2007), since they involve more elementary processes, such as representation, argumentation, idealization, generalization, etc.

The notion of semiotic function (SF) allows us to relate practices to the objects that are activated (Godino et al., 2007). An SF is a triadic relationship between an antecedent (initial expression/object) and a consequent (final content/object) established by a subject (person or institution) according to a certain criterion or correspondence code (Godino et al., 2007).

The theoretical tools just described allow for analysis of the mathematical activity in which: Firstly, temporal analysis of the mathematical practices carried out to solve a certain problem is performed; then the configuration of primary objects that intervene in those practices is analyzed (which provides information on the elements or parts of this mathematical activity), plotting the SF that interlinks the primary objects which intervene in mathematical practices (e.g., Breda et al., 2021); and finally, analysis in terms of processes is carried out again, to complete the analysis in terms of practices (which provides information on the temporal dynamics of mathematical activity).

**2.3 Networking between Extended Theory of Mathematical Connections and the Onto-semiotic Approach**

The networking of theories helps explore how different theories connect while respecting their conceptual and methodological principles to analyze the complexity of mathematics teaching and learning (Kidron & Bikner-Ahsbahs, 2015; Prediger et al., 2008). Rodríguez-Nieto et al. (2022a) examine the networking of the ETC and OSA, addressing three key questions about mathematical connections from both perspectives. To answer these questions, the authors analyze central publications of both theories and apply the Networking Methodology (Drijvers et al., 2013; Kidron & Bikner-Ahsbahs, 2015; Radford, 2008). They identify mathematical connections using the ETC framework and analyze them with the OSA. A key concordance is that both theories use content analysis. However, ETC applies a predefined typology of connections, while OSA employs diverse tools. The data were analyzed in terms of practices, primary object configurations, and SFs (OSA), which were later encapsulated as types of connections (ETC). Despite differing levels of detail, both theories complement each other, allowing for a deeper analysis. OSA reveals connections as part of a larger structure of practices, processes, and objects, providing insight into their function. This approach will also be applied to analyze students' mathematical productions.

**2.4. Categories of neuro-mathematical connections**

In this research on neuro-mathematical connections, it is essential to understand key concepts from neuromathematics and neuroscience. For example, word and symbol recognition is defined as a multimodal linguistic skill that involves understanding, combining, and using diverse forms of communication and expression (gestures, images, technologies, and other media) beyond spoken language, with the aim of improving mathematics teaching (Banegas, 2023).

Visual perception, spatial skills, and motor coordination are three interrelated abilities that play an essential role in students' learning and academic performance (Pahmi et al., 2024; Sudirman & Alhadari, 2020). Visual perception refers to the human ability to process and understand visual information from the environment. Spatial skills, on the other hand, are linked to the ability to interpret the position, orientation, and movement of objects in space, which is evident in activities such as reading and writing. Finally, fine motor coordination involves the ability to synchronize hand and eye movements to perform tasks that require precision, such as writing or drawing (Gutiérrez & Neuta, 2015; Macías & Cuellar, 2018; Narváez-Rumié et al., 2019; Price & Henao, 2011).

The association of mathematical concepts and formulas is defined as the ability to connect abstract ideas with mathematical expressions that represent them, allowing for the quantitative and precise description and analysis of phenomena. This facilitates the integration of ideas, variables, and values to solve problems and describe relationships in an organized and systematic manner (Godino et al., 2003; Mora, 2003). Intermediate calculations and unit conversions are mathematical operations performed during problem or equation solving, which can include unit changes, arithmetic operations, and the application of mathematical formulas (Arfken, 1985; Serway, 1990). These intermediate calculations contribute to a deeper understanding of concepts and the obtaining of precise results.

On the other hand, step-by-step problem solving and process understanding involve breaking down complex mathematical problems into simpler steps to facilitate their understanding and solution. Mayer and Moreno (2003) support the effectiveness of this approach, demonstrating that it reduces cognitive load and improves the understanding of mathematical concepts. Regarding verification, this consists of the process of confirming whether a statement or solution to an operation or problem is correct or incorrect (Azcárate & Amacho, 2003), while the conclusion represents the final result of a process that is reached through logical and systematic reasoning based on premises, definitions and established theorems (Lakatos, 2015).

## 3. Method

This research has a descriptive qualitative approach (Tenny et al., 2021), developed in three fundamental stages to explore neuro-mathematical connections. Stage 1: selection of volunteer participants. Stage 2: data collection through a problem situation, participant observation, and a written questionnaire. Stage 3: data analysis using the tool developed by Rodríguez-Nieto et al. (2024) and detailed in Cantillo-Rudas et al. (2024). One of the main advantages of qualitative research is its ability to understand and describe processes and patterns of human behavior that are difficult to measure numerically. Aspects such as experiences, attitudes, and behaviors are often difficult to accurately represent using quantitative methods, whereas a qualitative approach allows participants to express, in their own words, how, why, or what they were thinking, feeling, or experiencing at a specific moment or during a relevant event (Tenny et al., 2021).

### 3.1. Participants and context

Because the literature reports an ongoing problem regarding the difficulties students and teachers face in solving problems involving plane and three-dimensional geometric figures, participants from different educational levels were selected for this article. The first participant was a 29-year-old secondary school teacher (P1) with 10 years of work experience. Another participant was a 16-year-old high school student (P2) in eleventh grade according to the Colombian educational system. P1 and P2 attended a public institution in a municipality in the department of Atlántico, Colombia. Ten other participants (P3, P12) enrolled in an algebra and trigonometry course developed by the author of this article at a public university in Barranquilla, Colombia. It should be noted that all participants in this study were informed about the educational nature of this research and that it was not for financial gain.

### 3.2. Data collection

Given the qualitative nature of this study, data were collected using three instruments: first, an original problem was created (Cai & Ding, 2015) considering geometric aspects that students require to understand 2D and 3D figures, see the following problem situation:

> *Samuel went to the market and in the trash he found a box of tomatoes and a box of Oral-B toothpaste and used them in his Geometry class. Teacher Camilo asked him to find the volume of both boxes. What procedures did Samuel use to respond to his teacher? Measurements of the Oral-B box (toothpaste): width a = 3 cm; height b = 4.5 cm and length c = 20 cm; measurements of the tomato box: width a = 28 cm; height b = 37 cm and length c = 47 cm.*

Second, non-participant observation was conducted on teacher P2 and student P1 as they solved the problem on the blackboard in a secondary school classroom. Third, the problem was presented in the form of a questionnaire to the university students, who solved it using pencil and paper.

### 3.3. Data analysis

The analysis methods of these two theories are common and the analysis method used in the ETC was practically used, nourished with the OSA tools that allow a more detailed analysis of the mathematical activity (see Figure 3). The only indicators for identifying neuro-mathematical connections are the theoretical basis presented in section 2.4 and the fundamental input of the categories of mathematical connections.

Volume 2, Issue 1, pp. 58-80, E-ISSN: 3047-9207
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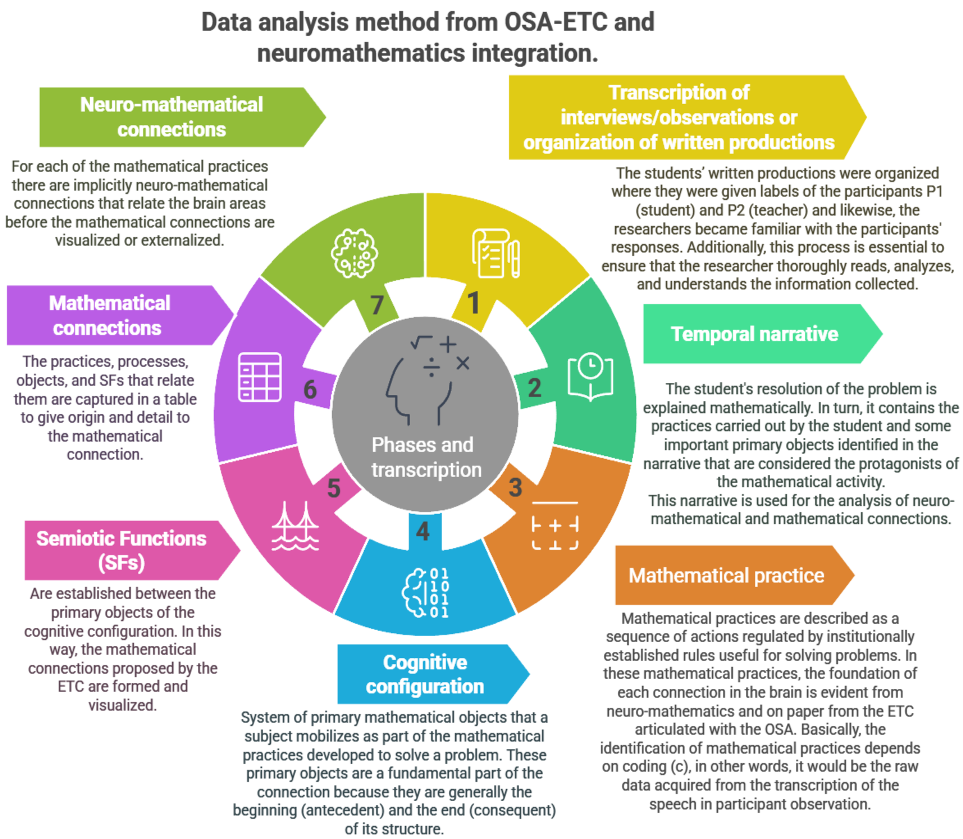
Figure 3
*Data analysis method from OSA-ETC and neuromathematics integration*




## 4. Results and Discussion

### 4.1 Results

#### 4.1.1 Temporal narratives of the student and teacher

These narratives reflect a summary of the mathematical activity carried out by the participants where the primary and secondary objects intervene (see Figure 4).

Figure 4
*Narratives of participants P1 and P2*

**Teacher's Narrative**

The problem was proposed by P1 who read and understood the problem and told the students that they were going to work with the volume of the tomato box that can be found considering the following measurements. Then, P1 drew the box on the board according to the established measurements and with the shape of a parallelepiped. Later, he mentioned that they should apply the volume formula equal to $v = a * b * c$. Next, he substituted the values of $a$, $b$, and $c$ into the formula $v = (28cm)(37cm)(47cm)$ and multiplied it considering the associative property $28cm * 37cm$ resulting 1.036 $cm^2$ and he multiplied that value by 47 cm, obtaining 48.692 $cm^3$. Finally, P1 stated that 48.692 $cm^3$ is the volume of the tomato box and the parallelepiped simultaneously.

**Student's Narrative**

The teacher (P1) proposed a problem to find the volume of a box, which was read and understood by P2. Then, P2 draws the box on the board considering its measurements in centimeters and in the shape of a parallelepiped. Subsequently, to find the volume P1 consider the formula $v = a * b * c$ and assume that $a$ is the base (width) equal to 3cm, $b$ is equal to the height $b$=4.5 cm and $c$ is equal to 20 cm which is the depth. P2 uses the associative property and multiplies $3cm * 4.5cm$ obtaining as a result $13.5cm^2$ and, finally, multiply $13.5cm^2$ by 20 cm to get the volume of the box equal to 270 $cm^3$.

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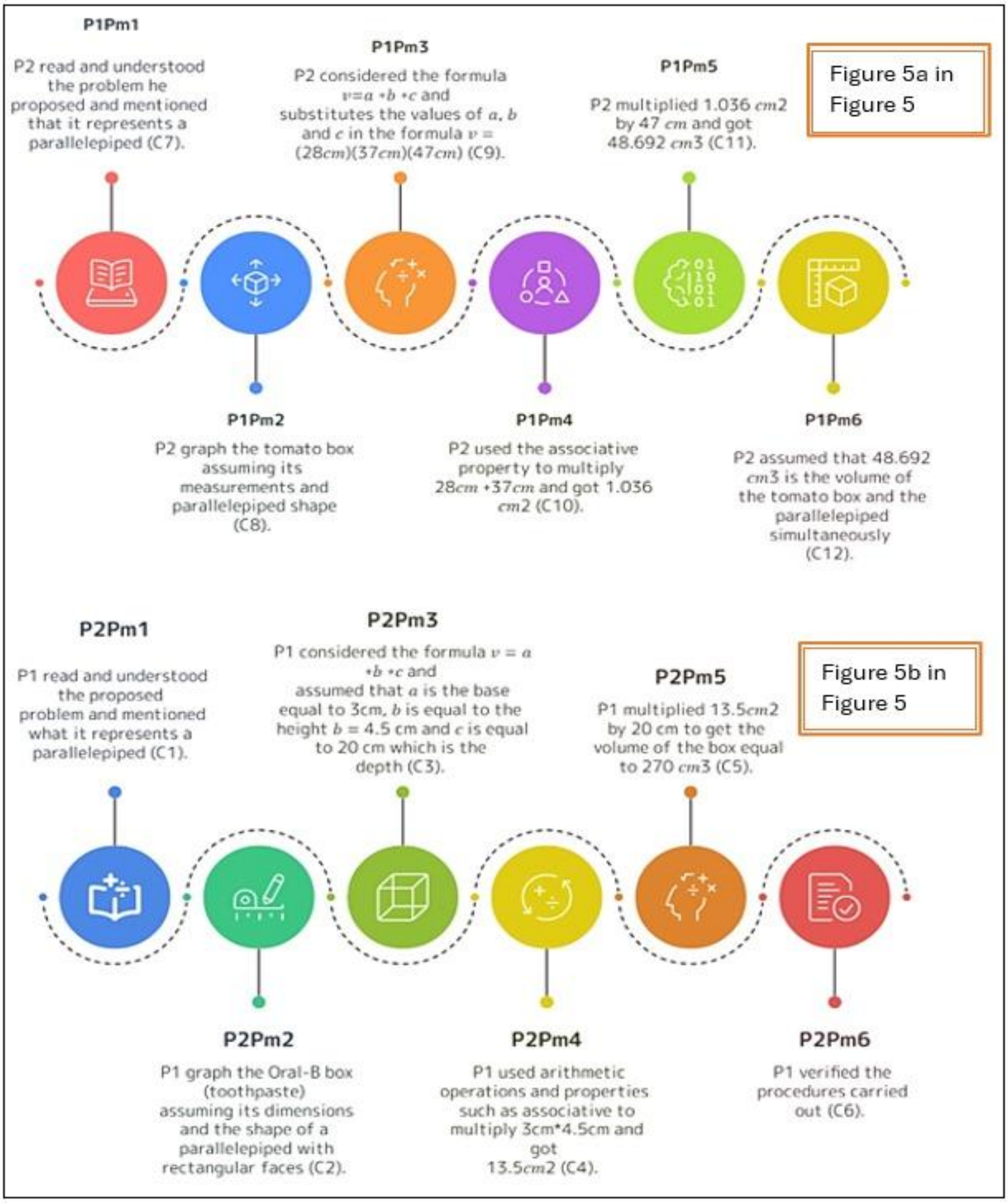
**4.1.2 Mathematical practices system**

Describing mathematical practices is one of the most important challenges in Mathematics Education, due to the diversity of individuals who engage in different mathematical activities. This is also due to the multiplicity of existing problems, but in this case, mathematical practices are analyzed within a specific, integrative theoretical framework: OSA articulated with ETC and Neuromathematics. The proposed problem was solved in parts: first, P1 found the volume of the tomato box (see Figure 5a in Figure 5) and then, P2 found the volume of the toothpaste box (see Figure 5b in Figure 5).

Figure 5

*Mathematical practices of P1 and P2*




It is worth noting that Figure 5 is a clear example of the sequenced steps that participants used to solve the proposed problem.

International Journal of Didactic Mathematics in Distance Education
Volume 2, Issue 1, pp. 58-80, E-ISSN: 3047-9207
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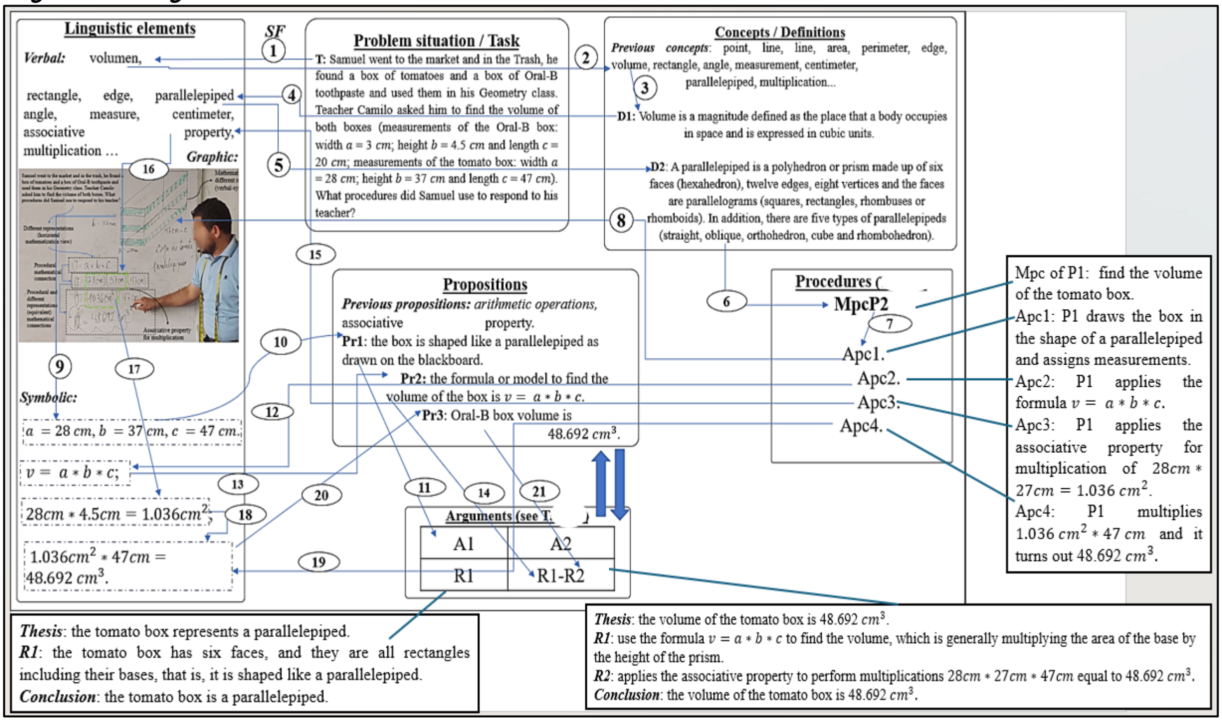
### 4.1.3 Cognitive configuration of primary objects and semiotic functions

In section 4.2. The sequenced actions carried out by P1 to solve the problem posed were evident. In mathematical practices, primary objects (PO) emerge that, connected to each other, make up networks of personal or cognitive objects. Figure 5 shows the mathematical practices carried out by P1 and P2 to solve the problem posed. These actions reflect a structured and systematic approach to problem-solving, demonstrating how they move through different stages of reasoning and decision-making. As part of these mathematical practices, various primary objects (POs) emerge. These are fundamental elements such as concepts, operations, or problem-solving strategies that serve as building blocks in the cognitive process. When these primary objects interact and connect with each other, they give rise to more complex structures known as personal or cognitive object networks. These networks represent the individual's internalized understanding, where multiple concepts, heuristics, and learned strategies intertwine to form a more complete mathematical framework. Figure 6 illustrates how these connections manifest in the cognitive configuration, highlighting the interrelationships between primary objects and the broader cognitive structures to which they contribute through semiotic functions, ultimately shaping P1's approach to mathematical problem-solving.

Figure 6

*Cognitive configuration of P1*




Subsequently, the configuration of P2 is shown, who follows similar procedures to P1, but with different measurements (see Figure 7).

*International Journal of Didactic Mathematics in Distance Education*
Volume 2, Issue 1, pp. 58-80, E-ISSN: 3047-9207
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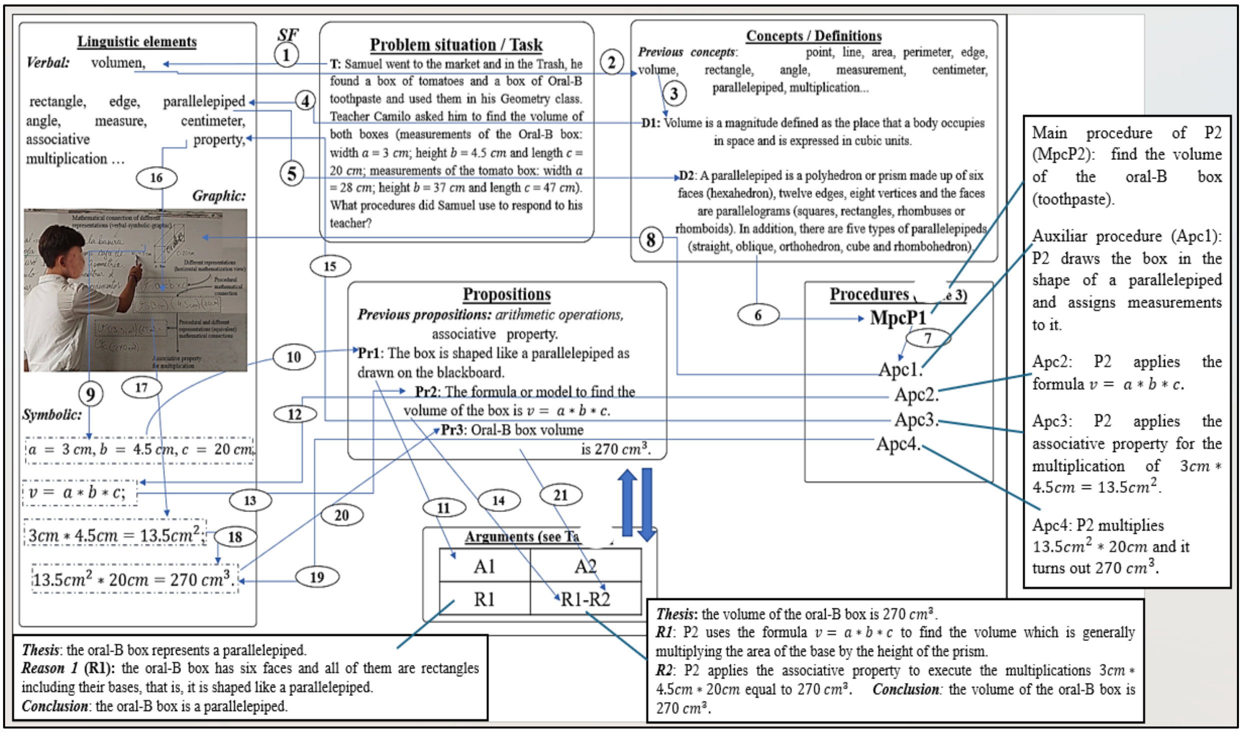
Figure 7
*Cognitive configuration of P2*




Figures 6 and 7 present a conceptual map that explains how Samuel calculated the volume of the toothpaste and tomato boxes shaped like a parallelepiped. It also presents the problem situation, where he is asked to find the volume using the given dimensions. Key definitions, such as volume and parallelepiped, are included, and the volume formula is established. The procedure is then detailed: recognizing the shape, assigning measurements, and applying the formula in two steps to obtain the result in cubic units. Furthermore, the figures reflect the use of tools from the onto-semiotic configuration, especially the semiotic functions (arrows with antecedent and consequent) that connect the primary objects and guide the understanding of the process.

Figure 8 presents the mathematical connections uniformly for P1 and P2, since, although the boxes have different dimensions, they correspond to the same type of parallelepiped. Furthermore, the proposed task has a high geometric potential, as it allows for the exploration of spatial properties and fundamental metric relationships. The cognitive configurations of P1 and P2 reveal similarities in the procedures used to solve the problem, in accordance with what was established in class, highlighting the use of strategies such as dimensional analysis, volume comparison, and the application of geometric formulas to justify their answers.

It is worth noting that Figure 8 shows mathematical connections based on the mathematical practices of Figures 5, 6 and 7, where the mathematical practices, processes, objects and semiotic functions are specified.

*International Journal of Didactic Mathematics in Distance Education*
Volume 2, Issue 1, pp. 58-80, E-ISSN: 3047-9207
https://jurnal.ut.ac.id/index.php/ijdmde

Figure 8
*Mathematical connections*

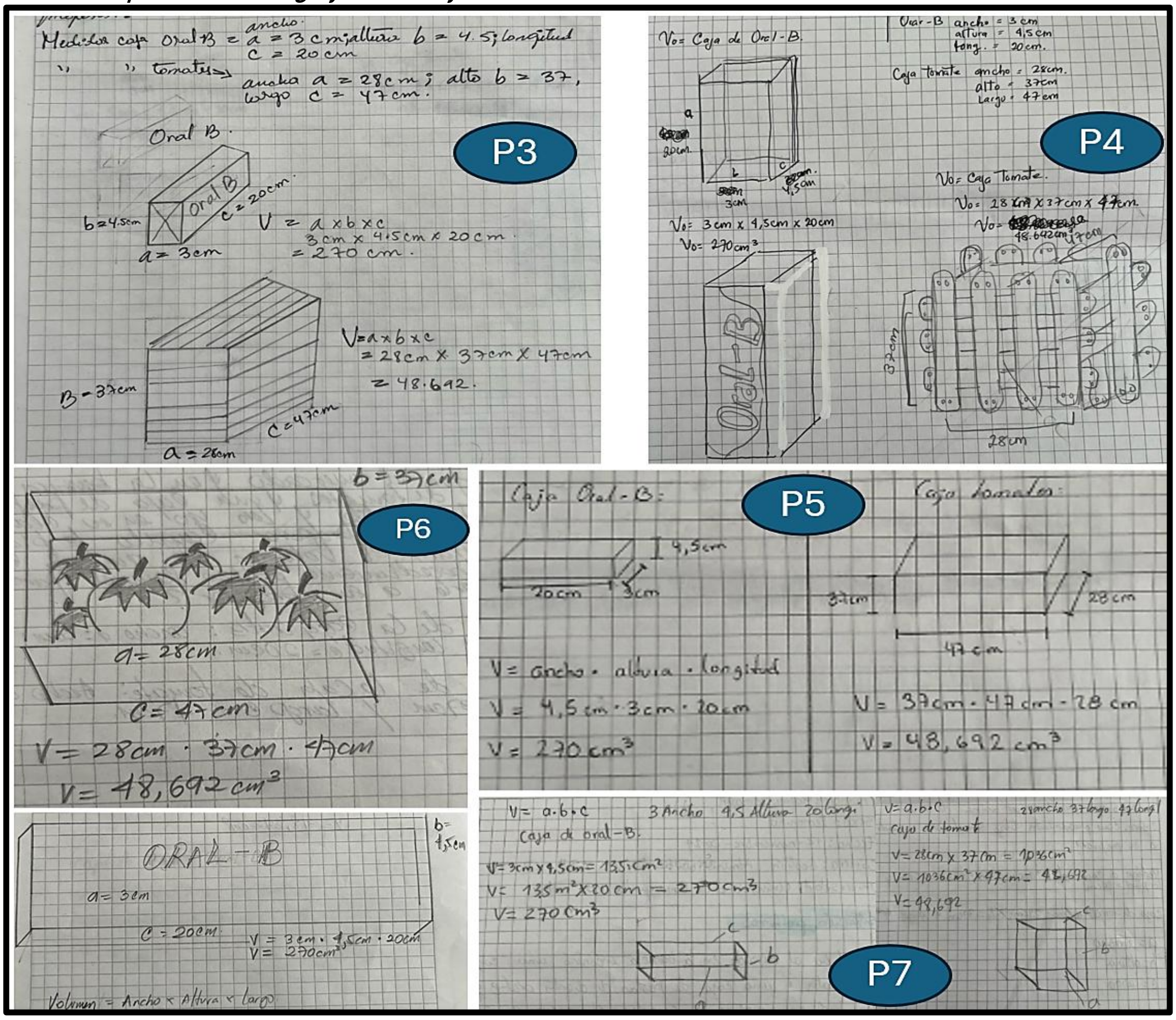
| Mp | Processes | Hyper-processes | Objects | SFs | Mathematical connections (ETC) |
|---|---|---|---|---|---|
| **Mp1** | -Signification/ Understanding. -Problematization. -Enunciation. | Problem-solving. | P1 and P2 read the problem and give meaning and use to the concept of volume and parallelepiped. | SF1 SF2 SF3 SF4 SF5 | Meaning Feature |
| **Mp2** | -Enunciation. -Idealization. -Representation. -Particularization. | Problem-solving. | P1 and P2 graphically represent the parallelepiped on the board associating measurements with it. | SF6 SF7 SF8 SF9 SF10 SF11 | Feature Different representations (alternate) Procedural Part-whole |
| **Mp3** | -Representation. | Problem-solving. | P1 and P2 used the model or formula $v = a * b * c$ and associated the measurements to find the volume. | SF12 SF13 SF14 | Feature Different representations (horizontal mathematization view) Procedural |
| **Mp4** | -Representation. -Algoritmization. | Problem-solving. | P1 used the associative property to multiply $28cm * 37cm$ to get $1.036\ cm^2$. P2 did the same procedure. | SF15 SF16 SF17 | Part-whole Different representations (equivalent) Feature |
| **Mp5** | -Algoritmization. -Representation. -Argumentation. | Problem-solving. | P1 multiplies $1.036\ cm^2$ by 47 cm and obtains $48.692\ cm^3$. P2 did the same procedure. | SF18 SF19 SF20 SF21 | Procedural Different representations (equivalent) |
| **Mp6** | -Argumentation. -Signification. | Problem-solving. | P1 assumed that $48.692\ cm^3$ is the volume of the tomato box and the parallelepiped simultaneously. For his part, P2 verified the procedures to ensure that the operations are well done. | SF21 | Meaning |

These tools not only enrich the study of connections but also reveal how cognitive processes intervene in the structuring of mathematical thinking. This new approach highlights that the development of each mathematical practice is closely linked to neuro-mathematical connections, highlighting the importance of cognitive processing in the construction of mathematical knowledge. The recent integration of ETC and neuromathematics (Cantillo-Rudas et al., 2024) allows for a deeper analysis, as it makes it possible to identify the neural connections activated in different areas of the brain when a person (P1 and P2) solves a mathematical problem. Furthermore, the previous analysis of mathematical connections with ETC already considered fundamental OSA tools, such as mathematical practices, processes, and objects, as well as SFs.

On the other hand, the students (P3, P4, P5, P6, P7, P11 and P12) from the university solved the problem in a similar way to P1 and P2, but made several different representation connections when they built the box plots (see Figure 9).

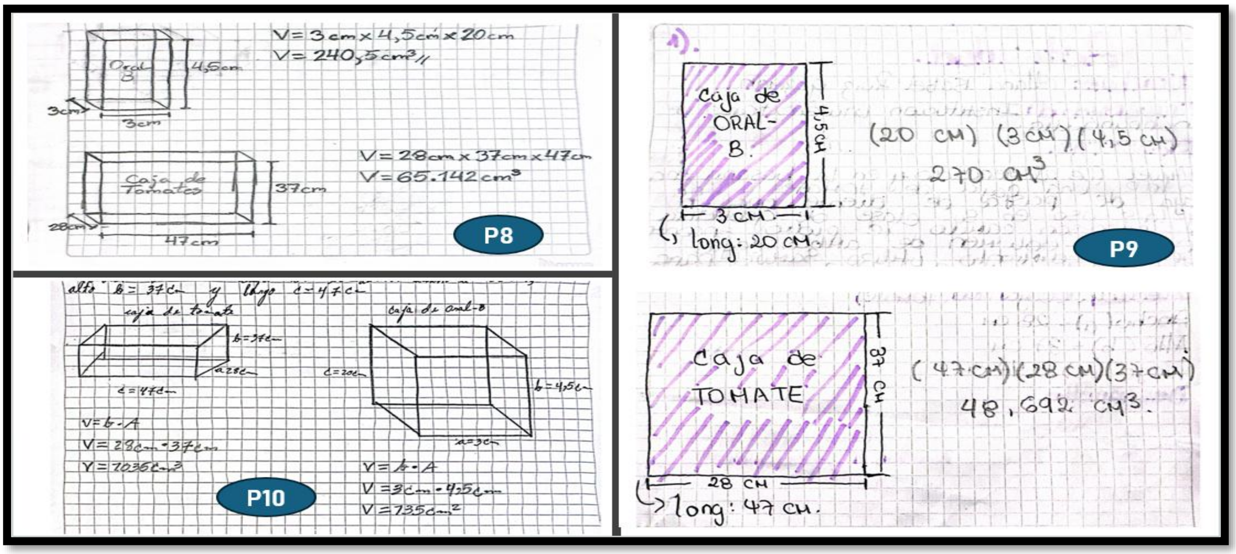
International Journal of Didactic Mathematics in Distance Education
Volume 2, Issue 1, pp. 58-80, E-ISSN: 3047-9207
https://jurnal.ut.ac.id/index.php/ijdmde

Figure 9
*Evidence of problem solving by university students*




However, just as there are students who consistently solve problems, there are also students who make mistakes caused by mathematical connections that have stopped being activated (see Figure 10).

Figure 10
*Evidence of errors caused by disconnections*



*International Journal of Didactic Mathematics in Distance Education*
Volume 2, Issue 1, pp. 58-80, E-ISSN: 3047-9207
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Figure 10 shows various errors made by students when solving the problem posed. In the case of P8, errors were identified both in calculating the volume and in the placement of measurements on the figure, evidencing confusion between width, length, and height. P9, in addition to making errors in the placement of measurements and displaying a lack of spatial awareness, constructed a cube instead of a parallelepiped and used an incorrect formula. Finally, P10 did not represent the figures in three dimensions, which could reflect a difficulty relating dimensions and volume.

**4.1.4 Neuro-mathematical connections**

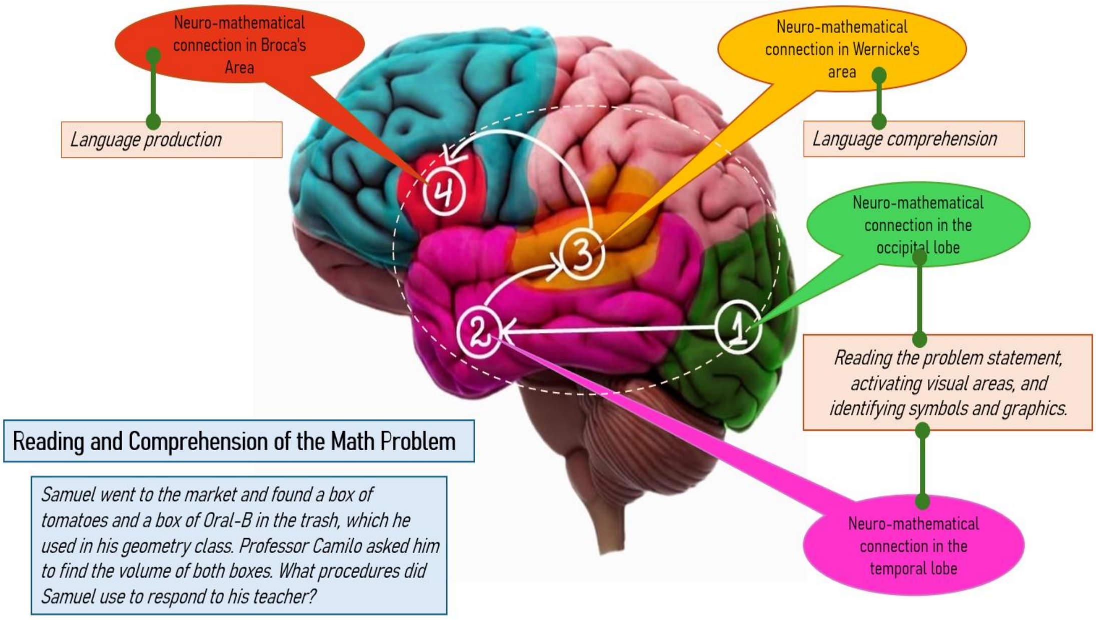
Neuro-mathematical connections were identified from theoretical and applied results evidenced in the literature on neuroscience and neuromathematics, which explain the activation of neural connections associated with mathematics (neuro-mathematical connections).

**4.1.4.1 Neuro-mathematical connection of term and symbol recognition (associated with P1mp1)**

As a student reads the problem situation, various brain areas work in a coordinated manner to process the information. First, the visual areas, located in the occipital and temporal lobes, are responsible for visual perception and symbol and word recognition. These areas allow the student to identify letters and written words. Furthermore, specific language areas are involved in the reading process. On the one hand, Wernicke's area, located in the posterior part of the superior gyrus of the temporal lobe (near the primary auditory cortex), is related to language comprehension. This area allows the student to understand the meaning of words and sentences. On the other hand, Broca's area, located in the frontal lobe of the left hemisphere (in most people), is associated with language production and the formation of grammatically correct sentences. In exceptional cases, especially in left-handed people, these language areas can be located in the right hemisphere. These two areas, Wernicke's and Broca's, are connected by a bundle of nerve fibers called the arcuate fasciculus, which facilitates communication between them. This connection is essential for processing the meaning of words and the relationships between them, allowing the student not only to decode the text, but also to interpret it and attribute meaning to it (Figure 11).

Figure 11

*Neuro-mathematical connection associated with mathematical practice 1*




**4.1.4.2. Neuro-mathematical connection of visual perception, spatial skills and motor coordination (P1mp2)**

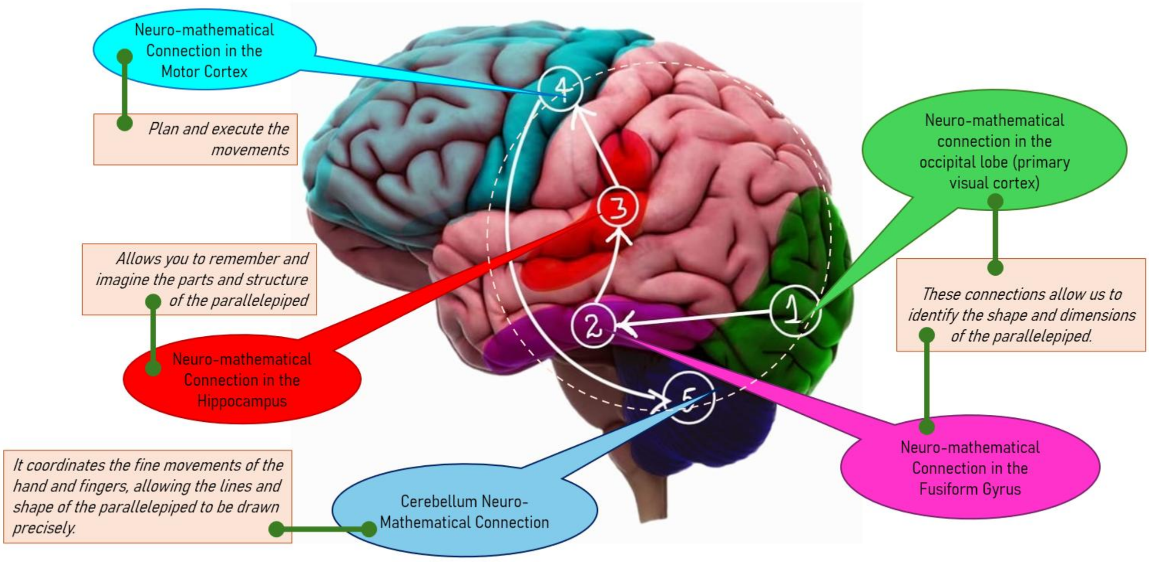
When a student draws a parallelepiped, neural areas related to visual perception, spatial skills, and motor coordination are activated. Visual perception allows the student to identify the shape and dimensions of the parallelepiped, beginning in the primary visual cortex, located at the back of the occipital lobe, where visual signals from the environment are processed. These signals are transmitted to the associative visual areas, and the fusiform gyrus, located in the temporal lobe, also involved in recognizing three-dimensional shapes and assigning meaning to the parallelepiped figure. Spatial memory, managed by the hippocampus,

International Journal of Didactic Mathematics in Distance Education
Volume 2, Issue 1, pp. 58-80, E-ISSN: 3047-9207
https://jurnal.ut.ac.id/index.php/ijdmde

is essential for remembering and imagining the parts and structure of the parallelepiped. Regarding motor coordination, the motor cortex located in the frontal lobe, which plans and executes movements, and the cerebellum, located in the back and lower part of the skull, which adjusts and coordinates the fine movements of the hand and fingers, allowing the lines and shape of the parallelepiped to be drawn accurately (Figure 12), are involved.

Figure 12

*Neuro-mathematical connection associated with mathematical practice 2*

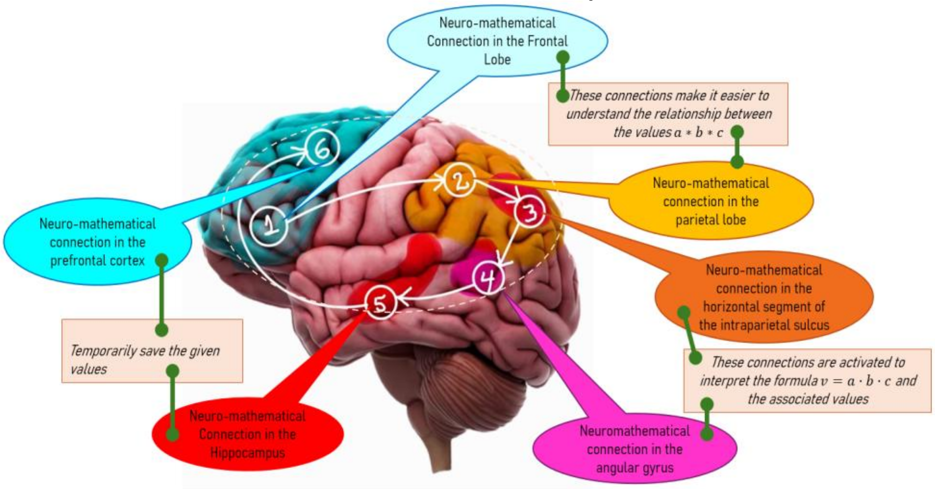



**4.1.4.3. Neuro-mathematical connection related to the association of mathematical concepts and formulas (P1mp3)**

This neuro-mathematical connection is activated when the student uses formulas expressed in symbolic language to understand the relationship between the values $v = a \cdot b \cdot c$ and their relationship to volume. Integration areas in the frontal and parietal lobes collaborate in this process: the frontal lobe assists with reasoning, while the parietal lobe handles spatial and numerical perception. Likewise, the horizontal segment of the intraparietal sulcus and the angular gyrus, associated with numerical and mathematical processing, are activated to interpret the formula $v = a \cdot b \cdot c$. The brain identifies that $a = 3\,cm$ represents the base or width, $b = 4.5\,cm$ the height, and $c = 20\,cm$ the depth or length. Furthermore, working memory, managed by the hippocampus and the prefrontal cortex, plays a key role by temporarily storing these values and their relationship to enable volume calculation (see Figure 13).

Figure 13

*Neuro-mathematical connection associated with mathematical practice 3*



International Journal of Didactic Mathematics in Distance Education
Volume 2, Issue 1, pp. 58-80, E-ISSN: 3047-9207
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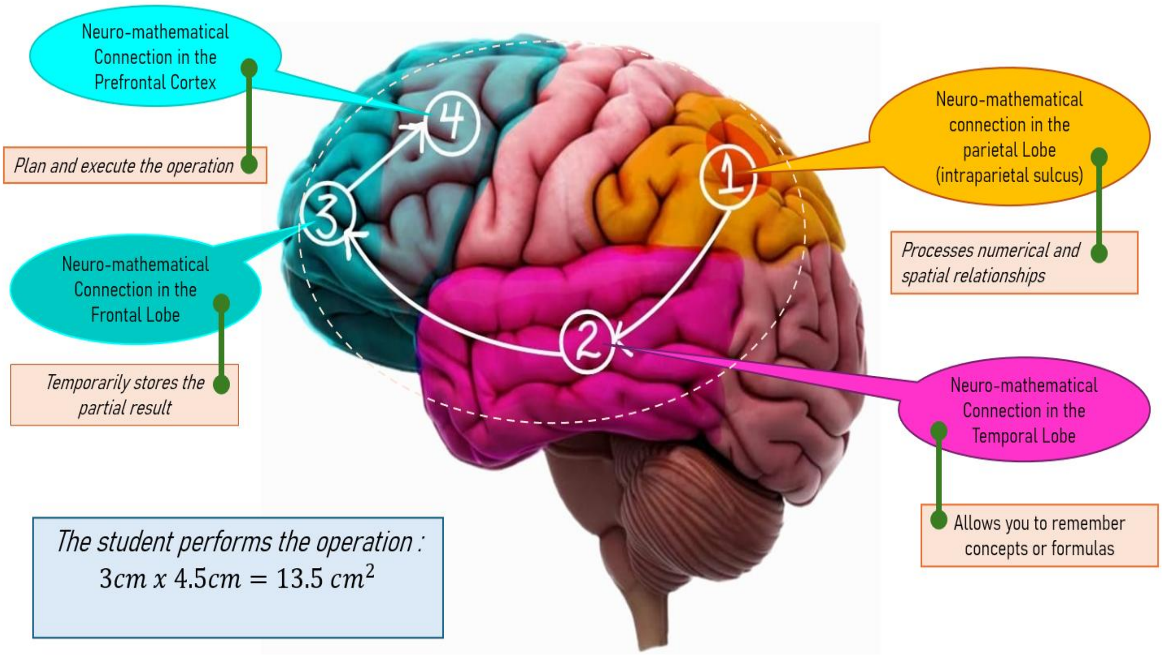
#### 4.1.4.4. Neuro-mathematical Connection of Intermediate Calculations and Unit Conversion (P1mp4)

This connection occurs when the student multiplies 3 cm × 4.5 cm. Their brain generates key neural connections to process the operation, which involves activating the neural networks associated with mathematical calculations, located in the parietal lobe, the temporal lobe, and the prefrontal cortex. The parietal lobe processes numerical and spatial relationships, while the prefrontal cortex is responsible for planning and executing the operation to obtain the result of $13.5\ cm^2$. Furthermore, the temporal lobe can intervene if the student needs to recall previously learned concepts or formulas, such as the relationship between units of measurement. Subsequently, the working memory, located in the prefrontal cortex, temporarily stores this partial result while the student performs the remaining operation to complete the calculation (see Figure 14).

Figure 14

*Neuro-mathematical connection associated with mathematical practice 4*

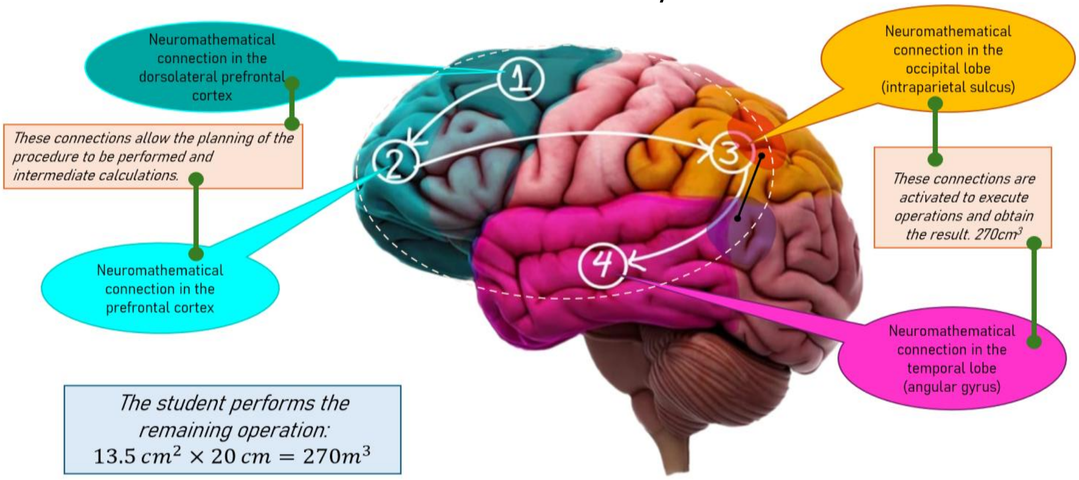



#### 4.1.4.5. Neuro-mathematical connection of step-by-step operation resolution and process understanding (P1mp5)

When a student solves mathematical operations step by step, such as multiplying $13.5\ cm^2 \times 20\ cm$, several brain areas are activated. In this step, the dorsolateral prefrontal cortex is activated to plan and organize the operation, integrating the partial result stored in working memory. During the multiplication operation, brain areas associated with mathematical calculations, such as the intraparietal sulcus and the angular gyrus, remain active to process the values and correctly execute the operation. These areas work in a coordinated manner, which facilitates understanding of the mathematical process and allows the final result of $270\ cm^3$ to be obtained, which represents the volume of the parallelepiped (Figure 15).

*International Journal of Didactic Mathematics in Distance Education*
Volume 2, Issue 1, pp. 58-80, E-ISSN: 3047-9207
https://jurnal.ut.ac.id/index.php/ijdmde

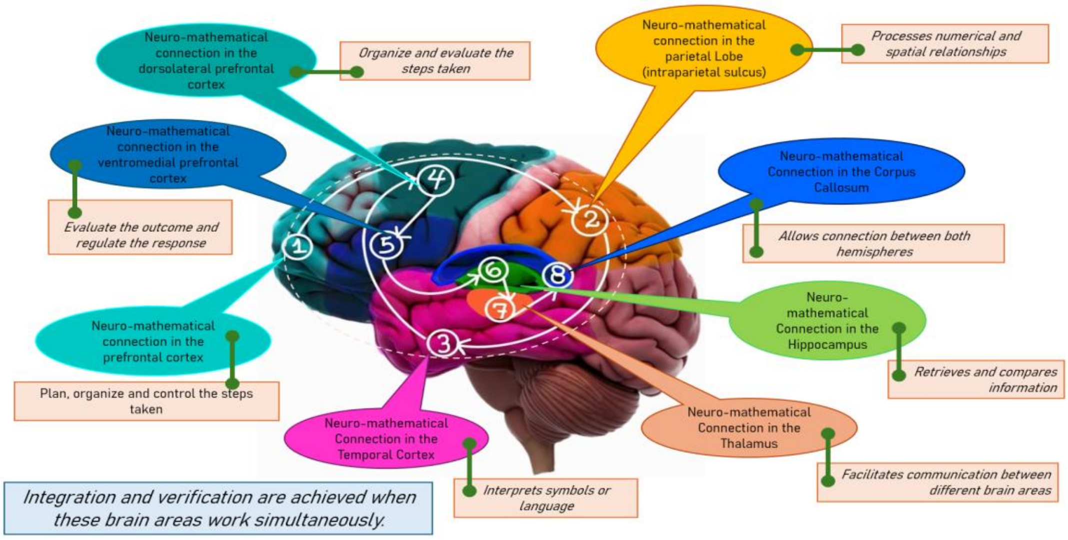
Figure 15
*Neuro-mathematical connection associated with mathematical practice 5*




#### 4.1.4.6. Neuro-mathematical connection of verification and conclusion (P1mp6)

Finally, the verification and conclusion connection is activated the moment the student completes the problem solution, which involves the participation of several brain areas to review and integrate the information. The prefrontal cortex plays a key role in this process, as it intervenes in the planning, organization, and control of the steps taken. In particular, the dorsolateral prefrontal cortex organizes and evaluates the steps taken, while the ventromedial prefrontal cortex evaluates the outcome and regulates the response. Likewise, the parietal cortex, especially the intraparietal sulcus, reviews numerical and spatial relationships. Other areas, such as the temporal cortex, may be involved in the interpretation of symbols or language, while the hippocampus helps retrieve and compare information to ensure that the result is coherent. Furthermore, the thalamus facilitates communication between different brain areas, and the corpus callosum enables the connection between both hemispheres, integrating symbolic and spatial aspects. This collaboration between the different brain regions allows the student to verify that the process and the result are correct (see Figure 16).

Figure 16
*Neuro-mathematical connection associated with mathematical practice 6*




### 4.2 Discussion

The present research presents that the interrelation between the theoretical perspectives ETC, OSA and neuromathematics allows not only to analyze mathematical connections externally on the blackboard or on pencil and paper (Rodríguez-Nieto & Font, 2025; García-García & Dolores Flores, 2021; Rodríguez-

Nieto et al., 2024), such as in the writing of operations, function graphs, but also to explore the internal processes that occur in the brain during mathematical problem solving. This approach opens new possibilities for the study of neuronal activity related to mathematical reasoning, contributing to the analysis of the cognitive phenomena involved in the learning and teaching of mathematics (Cantillo-Rudas et al., 2024).

Neuroscience is truly being used in this research to improve the analytical tools previously discussed, such as the ETC and OSA. That is, neuromathematics addresses more detailed aspects related to cognitive processes involving brain areas and neural connections. In fact, these types of results help teachers and students address the difficulties they have when solving mathematical problems, since errors occur because the necessary neuro-mathematical connection has not been made.

One of the key findings is the confirmation that mathematical connections are activated in similar patterns in both teachers and students when solving geometric problems. Specifically, it was observed that performing mathematical operations, such as multiplying measurements to calculate area, activates specific brain regions, including the parietal, temporal, and frontal lobes. These findings reinforce the idea that the analysis of mathematical connections should consider not only the symbolic and algorithmic representation of procedures, but also the way the brain processes and organizes information. Furthermore, this type of reflection corroborates the statements made by García-García and Dolores-Flores (2018) when they mentioned that connections are a cognitive process. However, within the broad research agenda on this topic, no work had been undertaken that delved into brain areas.

The results of this research further confirm and expand the Extended Theory of Connections (ETC) by demonstrating that mathematical connections are not only cognitive processes but also involve neural activation in specific brain areas (parietal, temporal, and frontal lobes). This provides a new perspective by incorporating a neuroscientific approach that validates the cognitive basis of ETC and demonstrates that mathematical connections are supported by specific neural processes. This research contributes substantially to the scientific literature because in the field of mathematics education, there are few studies related to neuroscience, and even fewer that explore neuro-mathematical connections. Furthermore, neuromathematics and difficulties in geometry have not received significant attention. This article strongly emphasizes analyzing the solution of a problem involving the volume of a box, highlighting the successful processes and the difficulties presented by the participants.

Another relevant aspect is the relationship between the errors made by students and the activation (or deactivation) of certain mathematical and neuro-mathematical connections. For example, a student who does not activate the procedural connection could be misusing a formula or showing difficulties in applying mathematical properties (Rodríguez-Nieto et al., 2024). This detailed analysis allows not only to identify errors but also to understand their underlying causes, which is crucial for designing more effective teaching strategies, specifically for geometry, which is an undervalued subject. Furthermore, it was identified that university students also participated in solving the task, although some showed difficulties in activating the procedural connections necessary to find the volume of objects representing parallelepipeds.

**Limitations**

Although this research reveals interesting results, it also has certain limitations. First, it focused exclusively on solving a geometric problem, leaving open the possibility that other types of mathematical problems activate different neuro-mathematical connections. Furthermore, although the use of previous studies and bibliographic references allows for a solid interpretation of the results, future research could benefit from the use of more advanced techniques, such as electroencephalography, to record neural activity in real time during problem solving.

## 5. Conclusion

This research provides empirical evidence that solving contextualized geometric problems activates a complex network of neuro-mathematical connections, involving cognitive, perceptual, and semiotic processes. The study revealed that students engaged a range of mental functions—such as symbolic recognition, spatial visualization, motor coordination, conceptual association, procedural reasoning, and reflective verification—while working through the task. These findings confirm that successful problem solvers demonstrated coherent and integrated neuro-mathematical connections, enabling them to transition smoothly between representations, apply relevant formulas, and make meaning of abstract concepts in context. In contrast, students who encountered difficulties exhibited fragmented or underdeveloped connections, particularly in associating visual models with algebraic expressions or maintaining procedural accuracy. These distinctions are critical for educators, as they highlight specific areas where instructional interventions can be designed to strengthen weaker connections and support deeper mathematical understanding.

Importantly, the study reinforces the value of an interdisciplinary framework combining the Onto-semiotic Approach and Connections Theory with insights from neuroscience. This integrated perspective not only enriches our theoretical understanding of mathematical cognition but also lays the groundwork for more effective pedagogical strategies that align with how students process and internalize mathematical concepts. Looking forward, future research should expand the investigation of neuro-mathematical connections across different mathematical domains and educational levels. Incorporating neuroscientific tools such as electroencephalography (EEG) and eye-tracking technologies will allow researchers to visualize and validate the real-time activation of connections. Such advancements could lead to the design of brain-informed instructional models that optimize learning outcomes in mathematics.

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**Author Contribution**
Author 1: Conceptualization, Writing - Original Draft, Formal analysis, and Methodology, Editing and Visualization.

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**Conflict of Interest**
The authors declare no conflict of interest.

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