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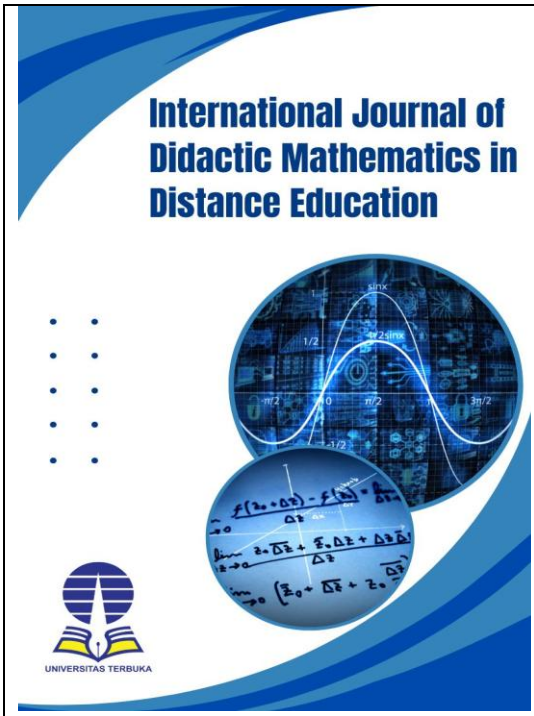

Hybrid didactic design research for fraction understanding during pandemic-induced remote learning: Addressing ontogenic, didactical, and epistemological obstacles

Muhamad Galang Isnawan[1*], Naif Mastoor Alsulami[2], Riyan Hidayat[3], R. Didi Kuswara[4]

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Hybrid didactic design research for fraction understanding during pandemic-induced remote learning: Addressing ontogenic, didactical, and epistemological obstacles

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Abstract
Understanding fractions remains a persistent challenge in mathematics education, particularly during emergency remote teaching conditions. While previous research has employed various methodologies to investigate fraction comprehension, few have utilized didactical design research (DDR) to systematically address multiple learning obstacles during pandemic-induced remote learning. This study contributes to the field by developing and validating a hybrid didactic design that integrates face-to-face and online learning modalities to overcome ontogenic, didactical, and epistemological obstacles in fraction education. Using a multi-stakeholder approach, we engaged 29 eighth-grade students (ages 14-18), 27 seventh-grade students (ages 13-15), one mathematics teacher, and 71 parents in a comprehensive DDR framework. The hybrid didactic design achieved content validity with a CVR value of 1.0 from seven expert validators. Through thematic analysis of learning obstacles and qualitative retrospective analysis of implementation data, we identified specific manifestations of each obstacle type and designed targeted pedagogical interventions. Post-implementation results demonstrated substantial reduction across all three obstacle categories. The key contribution of this research lies in demonstrating how hybrid didactic designs can effectively address multiple learning obstacles simultaneously through coordinated instructional strategies that leverage both synchronous and asynchronous learning environments. Our findings provide empirical evidence for the effectiveness of DDR methodology in emergency remote teaching contexts and offer a replicable framework for mathematics educators facing similar challenges. The study advances theoretical understanding of learning obstacles while providing practical solutions for fraction education in hybrid learning environments.

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Ontogenic obstacles.

1. Introduction

Previous studies reveal that fractions are a relatively important basic mathematical concept (Cortina et al., 2014; Getenet & Callingham, 2021). However, several previous studies, especially those related to the meaning of fractions, indicate that fractions are a problem for students (Harvey, 2012; Zhang et al., 2014). Some mathematics teachers do not know the meaning of the fraction itself (Isnawan et al., 2022; 2024). Several factors influence students' understanding of fractions, such as the teacher's knowledge of fractions (Adu-gyamfi et al., 2019), the learning designs that teachers use in learning (Wahyu et al., 2020), the complexity of the fractions themselves (Obersteiner & Tumpek, 2016), and students' knowledge of integers, including GCD and LCM values (Sun, 2019). Therefore, it can be concluded that fractions are a fairly important basic mathematical concept, but they are actually a problem for students which is influenced by teacher knowledge, learning design, and the complexity of the material.

Several researchers have used various research designs to examine the meaning of fractions in schools. A phenomenological design has been used by Isnawan et al. (2022) to examine students' perceptions on fractions. The study has revealed that most participants were limited to the meaning of fractions as ratios and quotients. The limited meaning is due to the limited knowledge of the teacher regarding the meaning of the fraction itself. A qualitative approach using clinical interviews has been used Kor et al. (2018) to identify the meaning of fractions in elementary schools. The results of this study have revealed that students with low and medium early math skills (EMS) do not have sensitivity to fractions. In contrast, students with high EMS have shown flexibility in interpreting and visualizing the shape of fractions. Bayaga and Bossé (2018) have used case studies to investigate how well elementary and middle school students understand fractions. The study concludes that students' understanding of concepts (meanings) and fractional operations is something that cannot be separated. Martinez and Blanco (2021) used a quantitative approach to assigning problem-posing assignments to ascertain the meaning of fractions in elementary schools, in contrast to earlier research. The research reveals that students tend to interpret fractions as part of a whole or part of a collection. Cramer et al. (2018) also used a number line to determine students' concepts of fractions using a quantitative approach and a quasi-experimental research type. The research reveals that students can use the number line model to convey ideas related to the meaning of fractions.

'Panic-gogy' is a learning condition caused by COVID-19 (Kamanetz, 2020). Learning, including mathematics, has shifted from face-to-face to online (Zhou et al., 2020). Mathematics comprises thinking and understanding (Harel, 2008), formed through mental actions prompted by problems (Suryadi, 2019b, 2019a). In learning, mathematics poses challenges (Gómez-Chacón, 2017). Due to factors outside of the student, such as learning, learning obstacles cause difficulties (Suryadi, 2019b). Learning obstacles include epistemological, didactical, and ontogenic obstacles (Brousseau, 2002). Job and Schneider (2014), Prabowo et al. (2022), and Siagian et al. (2022), ote encountering epistemological obstacles with limited context in learning mathematics. Didactical obstacles arise when educators face challenges in knowledge transposition (Suryadi, 2019b). Ontogenic obstacles occur when students lack interest or motivation (psychological), struggle to understand learned material (conceptual), or face technical or operational issues during learning (instrumental) (Isnawan et al., 2022).

One strategy used to minimize learning obstacles is implementing learning based on didactical situations (Brousseau & Warfield, 2020). In other words, preparing a didactic design is an alternative solution to overcome these learning obstacles. Didactic design is a learning design that is developed based on factors that cause students to experience learning obstacles. Didactic situations involve action formulation, validation, and institutionalization processes (Brousseau, 2002). Action-formulation occurs when a person solves a problem using a specific strategy (Prabowo et al., 2022). Validation is when individuals or groups draw conclusions about mathematical concepts obtained based on explanations of problem-solving strategies (Marfuah et al., 2022). Institutionalization happens when someone applies previously learned concepts in different contexts (Suryadi, 2019b).

Furthermore, didactic design takes various forms, including hybrid didactic design. The hybrid didactic design is a module combining ICT use for problem-solving with manual presentation of work results (Capinding, 2022; Sopacua et al., 2020). It involves activities organized based on didactic situations, integrating various forms of ICT, such as Zoom Meetings and learning videos (Prasetya & Mahmudah, 2021; Santagata et al., 2021; Yerushalmy & Olsher, 2020). This minimizes students' learning obstacles. The hybrid didactic design in this research refers to a design usable in online learning and limited face-to-face modes like home visits. It adopts activities from the epistemic learning pattern: initial activities with learning objectives, motivation, and exploring prerequisite material; core activities with didactic situations; and a final activity with reflection activities (Sukarma et al., 2024). The implementation of this hybrid didactic design is expected to optimize students' understanding of the meaning of fractions. Fractions have at least five meanings based on their use in life. They serve as part of a whole, act as a measure (e.g., ½ meter), function as ratios or comparisons, emerge as a result of division, and operate as operators (Isnawan, 2022).

Furthermore, there have been several other studies (Simon et al., 2018; Veloo & Puteh, 2017; Wahyu, 2021) that have aligned with the research described previously. However, few have used didactical design research (DDR) to help students interpret fractions. DDR is classified as relevant research when one wants to anticipate learning problems or obstacles in learning mathematics (Suryadi, 2019b). DDR seeks to identify sources of problems or types of learning obstacles that students experience and use these types of learning obstacles as a basis for preparing teaching materials or didactic designs (Marfuah et al., 2022; Suryadi, 2019a). Therefore, this study aims to describe a hybrid didactic design that can minimize students' learning obstacles in understanding fractions, especially during panic-gogy. The research questions arranged in order to achieve the research objectives are as follows:

a) What learning obstacles do students and parents experience in interpreting fractions during panic-gogy?
b) What is the form of a hybrid didactic design for learning the meaning of fractions during panic-gogy?
c) What obstacles are experienced during and after the implementation of the hybrid didactic design for panic-gogy fraction learning, particularly in students' interpretation of fractions?
d) What is the form of revision of the hybrid didactic design for fraction learning after implementation?

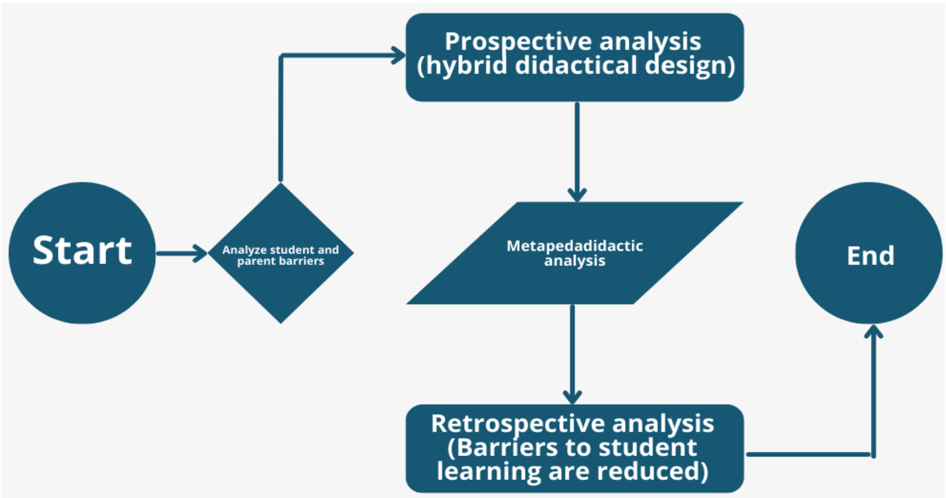
2. Method

2.1 Research Design

A qualitative approach was chosen in this study. The design in this research was DDR. DDR was chosen because it was relevant to the purpose of this study. First, the interpretive paradigm in DDR was related to the analysis of the impact of course design on students' way of thinking, which in this study was relevant to the analysis of factors that caused students to experience learning difficulties. Second, the critical paradigm in DDR tried to offer different ways to teach math, which in this case had to do with using a mix of different teaching methods to help students understand what fractions mean (Suryadi, 2019b, 2019a).

The procedure in this study followed DDR steps (Figure 1) (Marfuah et al., 2022; Prihandhika et al., 2022). First, prospective analysis: this analysis was a pre-learning analysis with two core activities, namely analyzing learning obstacles and compiling a hybrid didactic design. Second, metapedadidactic analysis: this was an analysis of how people learn with the main goal of using a hybrid teaching design to make sure that the connections, unity, and adaptability of teaching situations for math equations and inequalities are strong. Third, retrospective analysis was a reflection activity on the results of the implementation that aimed to revise the hybrid didactic design in accordance with the responses that students gave during lecture activities (Suryadi, 2019b, 2019a).

Figure 1
Research procedure



2.2 Participants

This research was conducted at junior high schools in West Lombok Regency, Indonesia. The junior high school was chosen because it was classified as a superior junior high school but had problems interpreting fractions. First, groups of students who had studied fractions with a total of 29 people. The age range of students was from fourteen to eighteen years. In addition, from the 29 participants, information was obtained that there were eight students with low EMS, fourteen students with medium EMS, and seven students with high EMS. EMS was measured using students' mathematics scores and the results of students' daily confirmation of mathematics learning by teachers. Students with math scores of less than 40 were included in the low category, students with scores ranging from 40 to 60 were in the medium category, and students with scores of more than 60 were in the high category (Veldman & Sanford, 1984). Most of the participants were female. Most participants were of Sasak ethnicity, and the rest were Balinese and Javanese. The participants came from various parental professions, such as entrepreneurs, farmers, mechanics, builders, laborers, civil servants, and police.

Second, groups of students who had not studied fractions at the junior high school level had a total of 27 people. The age range of the participants ranged from thirteen to fifteen years. Most of the participants were female. Most participants were of the Sasak ethnic group, and the rest were Balinese and Javanese. Most participants' parents were entrepreneurs and farmers, while the rest were construction workers, laborers, and civil servants. Third, a math teacher who taught fractions. The participant was around 26 years old and had five years of teaching experience. Fourth, parents of class VII students, totaling 71 people Participants had an age range from 35 to fifty years; 57 people were female and fourteen people were male; most worked as self-employed and freelancers, and the rest had professions as civil servants, military, and police. All student participants were asked to obtain a parental consent signature before providing data.

2.3 Data Collection

The main instrument in this research was the researcher, because this research used a qualitative approach (Creswell & Creswell, 2018). Several additional instruments in this study were tests of the meaning of fractions, guidelines for interviewing students and teachers, student biodata questionnaires, teacher biodata questionnaires, panic-gogy questionnaires for parents, hybrid didactic designs, observation sheets of learning implementation, and documentation studies. The interview used was an in-depth interview with several open-ended, semi-structured questions. The use of these interview techniques and questions allowed the informants to provide more comprehensive and varied responses (Brown & Danaher, 2017). Tests and student and teacher interview guides were used to obtain data related to factors that caused students to experience learning obstacles. To gather information about the difficulties parents encountered when implementing panic-gogy, parents filled out panic-gogy questionnaires. Hybrid didactic designs and observation sheets of learning implementation were used to obtain data related to student responses and learning implementation. Meanwhile, documentation studies were used as learning reflection material to determine the conditions of student learning obstacles and revise didactic designs.

Experts in mathematics education, pure mathematics, and psychology had tested the hybrid didactic design in this study for content validity. Four mathematics education experts with seven to nine years of teaching experience Two mathematicians with five to ten years of teaching experience. One psychologist with more than ten years of teaching experience. The researcher had visited all the experts with a hybrid didactic design draft to be read, given input, and given grades. Using the $CVR=((n_e-N/2))/(N/2)$ formula (Lawshe, 1975), the researcher then processed the value that the expert provided. n_e denoted the number of experts who gave an "essential" value, and N denoted the number of experts. Because all experts had given an "essential" value, a CVR value of one was obtained. The CVR value of the validity test was one so that it could be concluded that the hybrid didactic design had fulfilled the evidence of content validity (Lawshe, 1975).

2.4 Data Analysis

Data from several instruments was then analyzed using thematic analysis assisted by Nvivo-12 software to identify the learning obstacles students experienced. The thematic analysis steps included familiarizing oneself with the data (reading data repeatedly), compiling initial code, determining themes, reviewing themes, naming or defining themes, and compiling reports (Nowell et al., 2017; Sasidharan & Kareem, 2023). Nvivo-12 (Dalkin et al., 2020), in this study, helped in the coding process, especially when entering data into an initial code and the initial code into the theme it formed. Initially, data (student answer sheets and manuscripts of student and teacher interview results) were entered into Nvivo-12. After that, the researcher compiled the initial code from the data. In this step, triangulation of data sources (between students and between students and teachers) was carried out to strengthen the research findings. The next step was determining themes by grouping initial codes with the same characteristics into one theme. Nvivo-12 was very helpful when compiling the initial code and determining themes. Researchers dragged and dropped data into the initial code and used the same method to insert the initial code into the theme. After all the themes were formed, the researcher reviewed the existing themes by re-reading them and the initial code that formed them. This activity ensured that no initial code entered the theme that it should not. Next, the researcher gave a name or defined the theme according to its characteristics. Finally, the researcher compiled reports in various output forms, such as tables and descriptions. A mix of qualitative data analysis and retrospective analysis was also used to describe the hybrid instructional design and find a link between it and how it was used. This study's qualitative data analysis steps were data reduction, data presentation, and conclusion (Miles et al., 2014).

3. Results

3.1. What learning obstacles do students and parents experience in interpreting fractions during panic-gogy?

After conducting a thematic analysis of the data obtained, several themes were formed for each student's EMS. The full description of the themes can be seen in Table 1. Table 1 showed various forms of fraction meaning, such as fractions as rational numbers, means, integers, and parts of a whole. Meanwhile, one of the students' response excerpts can be seen in Figure 2. Figure 2 provided information that students interpreted fractions as rational numbers consisting of a numerator and a denominator. In other words, students basically understood that fractions were a representation of a rational number.

Figure 2
Excerpts of students' answers regarding the meaning of fractions

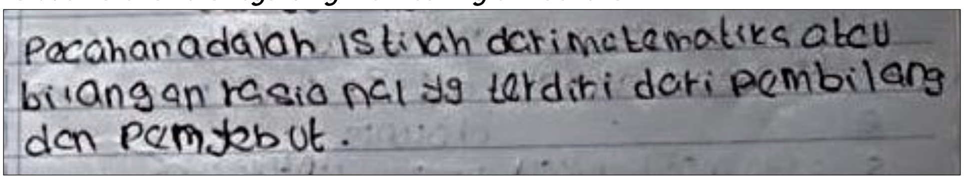


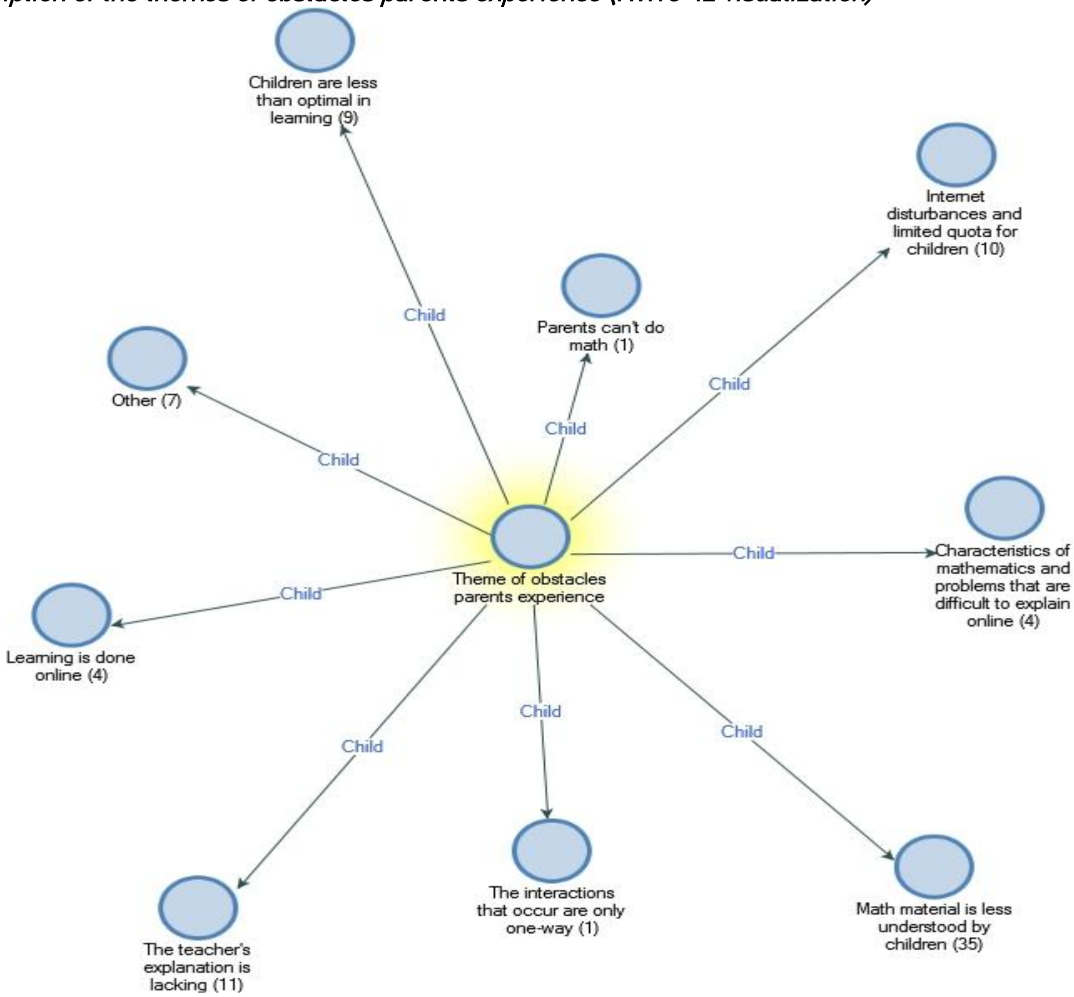
Figure 2 illustrates that students conceptualized fractions primarily as rational numbers composed of two essential components, namely a numerator and a denominator. This indicates that students recognized fractions as numerical representations rather than merely procedural symbols. Their responses suggest an initial understanding that a fraction expresses a relationship between two integers, where the numerator represents a part and the denominator represents the whole. Although the explanations were relatively simple, they demonstrate that students had grasped the foundational meaning of fractions as rational numbers. This basic conceptual understanding is important, as it serves as a prerequisite for developing more advanced fraction concepts, such as equivalence, comparison, and operations on fractions.

Table 1
Theme description before implementation

EMS Students	Theme Description	References
Low	Fractions as rational numbers.	14
	Fractions as a tool.	4
Average	Fractions as rational numbers.	26
	Fractions as a tool.	6
	Fractions as integers.	2
High	Fractions as rational numbers.	19
	Fractions as part of the whole.	2

Several themes were formed for the parent aspect, after the thematic analysis of parents' answers to panic-gogy questionnaires for parents. Figure 3 demonstrated that parents faced challenges relating to mathematical ideas as well as technical difficulties. According to at least 35 parents, their kids did not fully comprehend the math lessons that were taught during panic-gogy. Some examples were that students were unable to understand mathematics material well because the teacher's explanation was inadequate, and parents could not do mathematics, so they were unable to help students when solving problems.

Figure 3
Description of the themes of obstacles parents experience (NVivo-12 visualization)

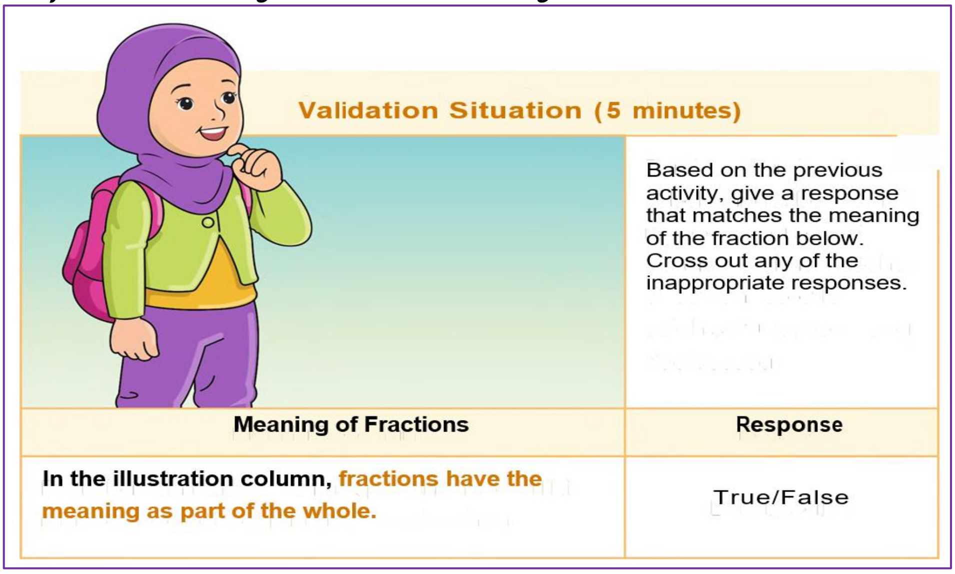


The figure illustrates (Figure 3) the central theme of obstacles experienced by parents in supporting their children's mathematics learning, particularly in an online learning context. This main theme is connected to several subthemes, including children's limited understanding of mathematical material, one-way interactions during learning, and teachers' explanations that are perceived as insufficient. Parents also face difficulties related to the characteristics of mathematical content and problems, which are challenging to explain through online media. In addition, technological issues such as limited internet quota and learning disturbances further hinder the learning process. Other contributing factors include parents' limited mathematical knowledge and children's suboptimal learning conditions. Overall, the figure indicates that these obstacles are complex and interrelated, involving pedagogical, technological, and familial factors that collectively affect children's mathematics learning experiences.

3.2 What is the form of a hybrid didactic design for learning the meaning of fractions during panic-gogy?

Based on the answers to the second research question, it was found that there were three types of learning obstacles, namely didactical, epistemological, and ontogenic obstacles. As previously described, the didactical obstacle was related to the teacher's limited knowledge of the meaning of fractions. Therefore, the hybrid didactic design offered a solution by integrating the various meanings of fractions into the learning context. Figure 4 provided an example of a design snippet related to the meaning of fractions arranged in a hybrid didactic design. Figure 4 showed that students were asked to validate whether a fraction had meaning as part of a whole or not.

Figure 4
A snippet of a hybrid didactic design related to the meaning of fractions



Regarding the ontogenic obstacle, which was instrumental in nature, the researcher arranged a design in the form of using various scaffolding to assist students in solving problems. Figure 5 presented an example of a snippet of a hybrid didactic design that contained scaffolding. The scaffolding used in Figure 5 was a number line model. The illustration given by the teacher was directed so that students used the number line model when solving the problem. Students were directed to divide the number line model into several parts in order to solve the problem correctly.

Furthermore, researchers used various contexts and forms of illustration to minimize learning obstacles with this type of epistemological obstacle. Figure 6 showed an example of the context or illustration model used to find the meanings of fractions. Figure 6 used a model of a collection of objects (oranges) as an illustration to help students solve problems. Students were expected to be able to easily solve problems when they were presented with an illustration of the collection of oranges.

Figure 5

A snippet of a hybrid didactic design containing scaffolding

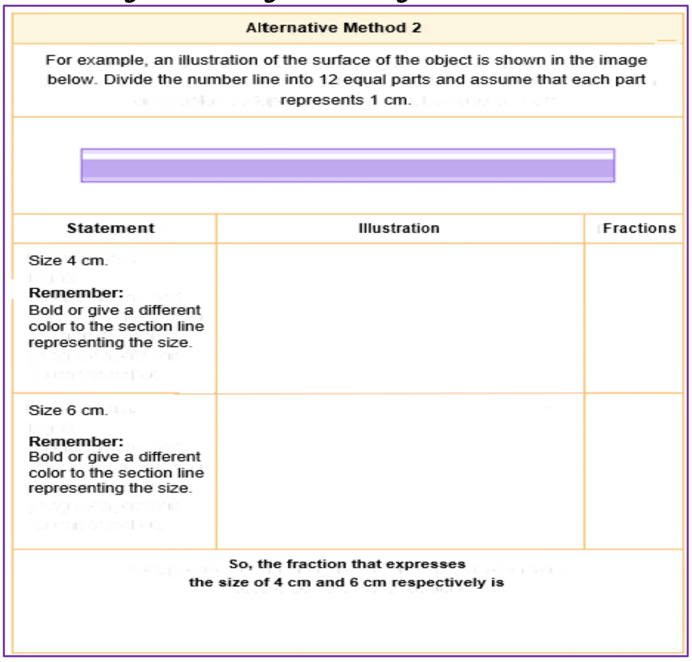


Figure 5 illustrates a snippet of a hybrid didactic design that incorporates scaffolding to support students' understanding of fractions as representations of length. The design uses a number line divided into 12 equal parts, with each part representing 1 cm, to visually connect measurement and fractional reasoning. Through step-by-step instructions and explicit prompts, such as highlighting or coloring the segment that represents a given size (e.g., 4 cm and 6 cm), students are guided to identify corresponding fractions of the whole. This structured support helps learners move gradually from concrete visual representations to abstract fractional concepts, demonstrating how scaffolding within a hybrid didactic approach facilitates conceptual understanding rather than mere procedural knowledge.

Figure 6

An example of a context or illustration model in a hybrid didactic design

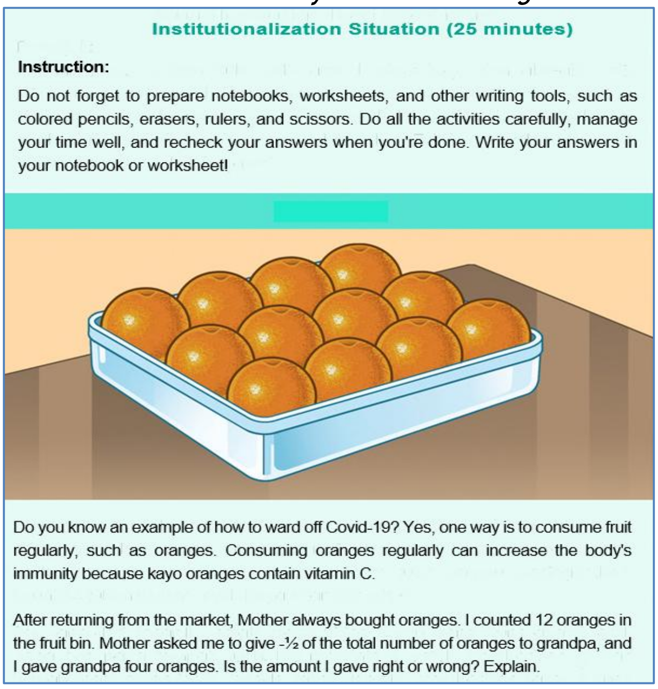


Figure 6 presents an example of a contextual or illustration model within a hybrid didactic design, where a real-life situation is used to institutionalize mathematical concepts. The context of buying and sharing oranges is employed to connect students' everyday experiences with the formal understanding of fractions, specifically the concept of one-half. By asking students to determine whether giving four oranges out of twelve represents one-half of the total, the design encourages reasoning, justification, and conceptual validation rather than rote calculation. This contextual model supports the transition from informal, experience-based reasoning to formal mathematical knowledge, illustrating how hybrid didactic designs integrate realistic situations as a bridge toward institutionalized mathematical concepts.

In general, the hybrid didactic design consisted of three learning activities, namely preparatory, lecture, and evaluation, which integrated various didactic situations. Preparatory was an initial learning activity consisting of *Let's Guess* and *Let's Read* activities. *Let's Read* aimed to confirm students' prerequisite knowledge about LCM and GCD, as well as several illustrative models needed in learning. *Let's Read* aims to attract students' interest or motivation to learn by providing stories related to the benefits of fractions in everyday life. The lecture included the activities *Let's Find Out*, *Let's Tell a Story*, and *Let's Summarize*. These three activities aimed to construct the meaning of fractions through problem-solving activities by students. The final activity consisted of the *Let's Practice* and *My Reflection* activities. *Let's Practice* aimed to use the concept of the meaning of fractions in solving problems in different contexts or situations. Meanwhile, *My Reflection* aimed to re-check students' understanding of the meaning of fractions, feelings or social emotions, and character commitment.

3.3 What obstacles are experienced during and after the implementation of the hybrid didactic design for panic-gogy fraction learning, particularly in students' interpretation of fractions?

Researchers usually start a *Zoom* meeting before the set schedule. At the beginning of opening the platform, researchers found several students already in the main room. While waiting for the other students, the researcher asked the students to prepare a hybrid didactic design and writing tools that previous researchers had shared. The researcher opened the learning activities by greeting and opening prayer. The researcher then asked the students to do *Let's Guess* activities. The researcher asked students to recall how to do the LCM and GCD they had learned at the elementary school level. LCM and GCD were two concepts in mathematics that were prerequisite material when learning fractions, especially when simplifying and adding fractions with different denominators. After that, there was a short conversation between the students and the researcher. Table 2 presents excerpts from student interviews with researchers.

Table 2
Snippet of the researcher's interview with the Let's Guess activity students

Researcher Questions	Student Answers
Can you recall how to find the Least Common Multiple (LCM)? Anyone still remember?	*HMS: I forget sir.*
	SL: Forget sir, forget (students laugh).
What is the LCM of 2 and 3?	*RAF: 12.*
	HMS: 13.
	SL1: 11 sir, 11.

Based on Table 2 and researcher observations during the implementation of the hybrid didactic design, information was obtained that most students needed to remember how to find the LCM value. Because there were no correct student answers, the researcher directed students to find the appropriate LCM values. After students seemed to understand how to find the LCM value, the researcher continued the learning activity by asking students what the GCD score was out of 4 and 6. In this activity, students also needed help determining the optimal GCD value. The researcher then explained by stating that the exact integers in each factor tree were the GCD values of the two numbers.

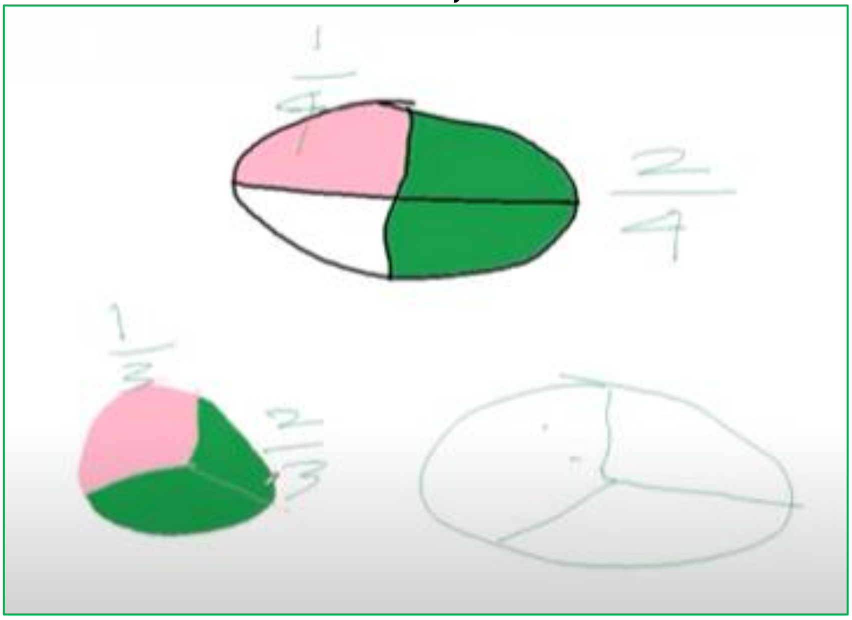
In connection with the *Let's Guess* Illustration Model activity, students could pair the illustration model with the appropriate integer. However, when students were asked about the reasons, they seemed unable to give a good explanation. Therefore, the researcher provided reinforcement related to the reasons that followed the answers given by students. The researcher continued learning through *Let's Read* activities. One student with ZA's initials volunteered to read a motivational story presented in a hybrid didactic design. After ZA finished reading, students with the initials SH and RAF conveyed the story's gist.

SH stated that fractions are often used in commerce and agriculture. Meanwhile, the RAF stated that quite a lot of mathematical material is used in everyday life.

The next activity the students did was *Let's Find Out*. In the breakout room (BOR), students worked in groups to complete this activity. Before entering the BOR, the researcher explained to the students how to do the *Let's Find Out* activity. The researcher described that students were allowed to answer their problems according to their abilities (alternative method 1). The researcher also explained that students could use other alternative methods with the help of several illustrative models so that problems could be solved. When the discussion activities began, students in group 2 needed help determining which students acted as note-takers. The researcher then appointed the RAF as the minutes to resume the discussion activities. For group 1, the researcher directed students to actively discuss with other group members. Whereas for group 3, the researcher gave a trigger question by directing the student group to divide the illustration model into several equal parts. The researcher then returned to group 2 and directed students to divide the model illustrations according to the group's agreement. In addition, the researcher also directed students not to push themselves if they could not solve the problem independently but to use other alternative methods. Finally, group 2 used alternative method 1. One of the students was seen leading the discussion activity.

Unlike the previous group, group 1 was able to find the final answer. Researchers directed students to determine students who would present the results obtained—only after the final minutes of *Let's Find Out* activity group 3 still had not finished their work. The researcher then directed the student group to use the second alternative to solve the problem on time. The researcher returned all the students to the main room a few minutes later. The next activity was *Let's Tell a Story*. HMS and RAF were the participants in this activity who represented group 1. HMS and RAF were able to convey their answers and were as expected. Group 2 and Group 3 also gave appropriate answers. Figure 7 then provided snippets of student answers for each group. Figure 7 provided information that there were three groups of students who made illustrations using a circular area model. Two groups divided the circular area model into three parts. One group only made an illustration but had not yet stated the fractional form (uncolored circle). Another group divided the circular area model into three parts, with two parts as $\frac{2}{3}$ (green) and one part as $\frac{1}{3}$ (pink). Meanwhile, the other group divided the circular area model into four parts, with the green part as $\frac{2}{4}$ and the pink part as $\frac{1}{4}$.

Figure 7
Snippets of student answers in the Let's Find Out activity



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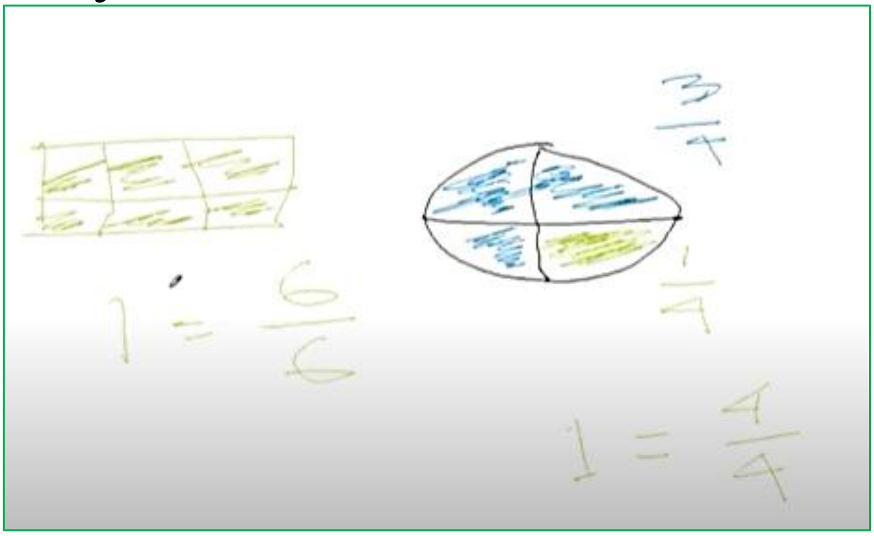
Students in the *Let's Summarize* activity seemed to be able to make conclusions that fractions have meaning as part of the whole and fractions as ratios or comparisons. In addition, the researcher provided reinforcement that fractions are part of a whole and also introduced another meaning of fractions, namely fractions as a whole from a whole. Figure 8 provided information regarding the meaning of fractions as a whole of a whole. One area model (rectangle) illustrated the fraction $\frac{6}{6}$ = 1. Another area model (circle) illustrated the fraction $\frac{4}{4}$ = 1.

Figure 8

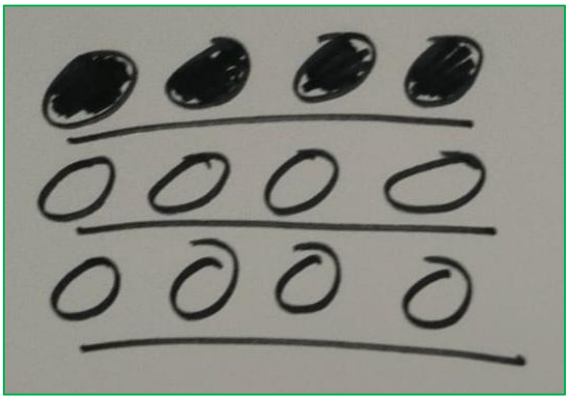
Snippets of the meaning of fractions as a whole of a whole



The next activity was *Let's Practice*. Students seemed to be able to provide answers related to problem 1 correctly. Snippets of students' answers to problem 1 are displayed in Figure 9. Students created an illustration in Figure 9 that addressed the orange problem in Figure 6. Students found it easier to choose the right illustration model when there was an orange model available. Students decided to shade the four oranges that were given to the grandfather using a collection of objects (small circles).

Figure 9

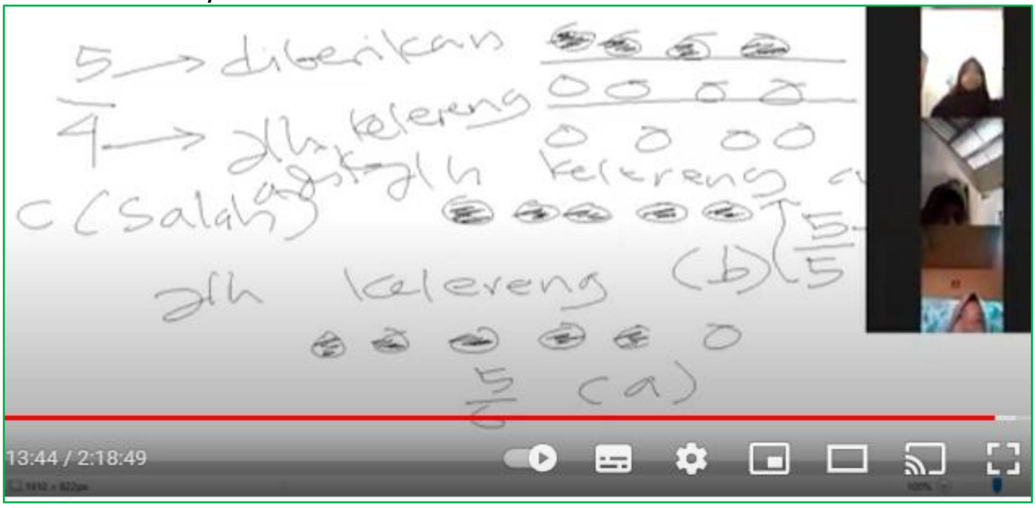
Illustration of answers to problems 1



In contrast to problem 1, none of the students could solve problem 2. Therefore, the researchers took the initiative together with the students to solve the problem. Figure 10 provided an illustration of the answer to problem 2. Figure 10 showed that students and researchers created an illustration of a collection of marble-shaped objects. The illustration provided the idea that the fractional form in the problem of sharing marbles could not have a denominator greater than the numerator. This was because the denominator represented the number of marbles given, while the numerator stated the number of marbles distributed. A child could not possibly distribute more marbles than the number he had. The last activity was *My Reflection*. Students in this activity had completed everything in that section. All students revealed that they enjoyed learning that day because they used various models of illustration and drawing when studying. Students reported understanding fractions primarily as part-whole relationships and as ratios, following the lesson. In addition, students revealed material they had yet to understand.

Figure 10

Illustration of answers to problems 2



Based on the results of the researcher's reflection, several weaknesses were found in the first meeting. These weaknesses included the need for the researcher to remember to convey learning objectives, a tendency to rush in giving trigger questions to students, suboptimal use of pen-tablets by the researcher, and audio recordings of one of the BORs having problems. Therefore, the researcher planned to convey the learning objectives at the next meeting, be slower when asking trigger questions, and optimize the use of pen-tablets during learning.

Futhermore, descriptions of themes related to learning obstacles that students experienced after implementing hybrid didactic designs during panic-gogy in mathematics learning are presented in Table 5. Table 5 also exposed some misconceptions about the meaning of fractions, such as fractions being tools and fractions being objects. In other words, there were indications that students had encountered learning difficulties.

Table 5

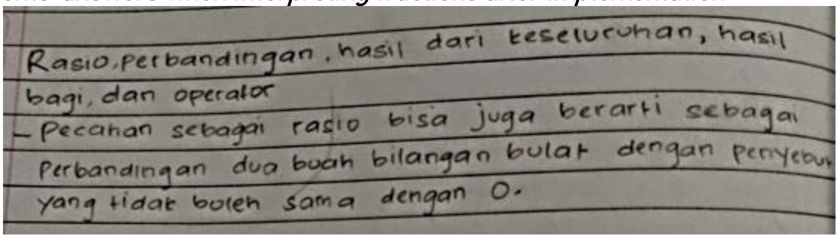
Description of the theme after the implementation of the hybrid didactic design

EMS Students	Theme Description	*References*
Low	Fractions as ratios.	9
	Fractions as a tool.	2
	Fractions as a measure.	1
	Fractions as operators.	1
Average	Fractions as ratios.	9
	Fractions as part of the whole.	3
	Fractions as quotient.	3
	The meaning of the fraction could not be identified.	3
	Fractions as a measure.	2
	Fractions as operators.	2
High	Fractions as ratios.	7
	Fractions as part of the whole.	5
	Fractions as a measure.	4
	Fractions as operators.	4
	Fractions as quotient.	4
	Fractions as objects.	1

Researchers had conducted interviews with students to unearth the factors contributing to students' learning obstacles, identify the types of learning obstacles students faced, and gain insights from students. One student with low EMS disclosed insufficient study time, making it challenging to answer questions during exams. Students reported that teachers assigned tasks from other subjects during the exam schedule. Several students with moderate EMS echoed similar sentiments, emphasizing the need for more study time to comprehend fractions as a whole. Students also provided reasons for high EMS, citing the burden of multiple assignments that hindered optimal performance in answering exam questions. These were students actively participating in class discussions. Figure 11 shows samples of students' responses to questions about fraction interpretation following implementation. Students were able to understand fractions as operators, quotients, comparisons, ratios, and parts of a whole, according to Figure 11. Indeed, students were able to explain that when a fraction is expressed as a ratio or comparison of two integers, the denominator must not equal 0.

Figure 11

Excerpts of students' answers when interpreting fractions after implementation



Rasio, perbandingan, hasil dari keseluruhan, hasil
bagi, dan operator
- Pecahan sebagai rasio bisa juga berarti sebagai
perbandingan dua buah bilangan bulat dengan penyebut
yang tidak boleh sama dengan 0.

3.4 What is the form of revision of the hybrid didactic design for fraction learning after implementation?

Based on the previous description, it was found that most of the responses given by students during the implementation of the hybrid didactic design aligned with the didactic-pedagogical expectations outlined in the design. However, some editorial or trigger questions required correction. In this activity, it was revealed that the researcher had initially miscalculated the LCM values of 2 and 4. Both researchers and students acknowledged that the correct LCM value for 2 and 4 is 4, not 8 as originally indicated. Consequently, the researcher revised the hybrid didactic design, directing students to recognize that the correct LCM value for 2 and 4 is 4.

Concerning the *Let's Read* activity, no significant revisions were made regarding the Motivational Stories. However, it was expected that after reading, students should be able to convey the essence of the reading. Space or instructions to express the essence of the story were not initially included in the hybrid didactic design. Therefore, the revision of the hybrid didactic design aimed to provide a separate space or column after the Motivational Story as a platform for students to convey ideas related to Motivational Stories. Meanwhile, for the *Let's Find Out* activity, there was a modification in the illustration model used. This revision was prompted by the initial illustration model, which tended to resemble two circular areas (inner circle and outer circle). In contrast, the intended illustration model was meant to depict the shape of fried rice.

4. Discussion

Based on Table 1, information is obtained that fractions as rational numbers are quite dominant in this study. When compared with theories related to the meaning of fractions (Dewi et al., 2016; Martinez & Blanco, 2021), for students with high EMS, information is obtained that there is no wrong meaning of fractions. Meanwhile, for students with moderate and high EMS, information was obtained that there were some erroneous meanings of fractions, namely fractions as tools and fractions as integers.

The initial code that is quite dominant in forming the meaning of a fraction as a rational number is a fraction as two numbers consisting of a numerator and a denominator. The results of this study are in line with (Mamonto et al., 2018) which revealed that students tend to interpret fractions as a notation consisting of a numerator and denominator. Fractions as part of the whole then tend to be quite difficult for students to explain in this study. This is possibly due to the level of cognitive development of junior high school students which tends to be classified as formal operations (Isnawan et al., 2022). The results of this study are in line with several previous studies which revealed that students tend to have difficulty interpreting fractions as part of a whole (Fauzi & Suryadi, 2020b; Villalba et al., 2019). Unlike the previous ones, the results of this study contradict several previous studies which revealed that students tend to be able to interpret fractions as part of a whole (Getenet & Callingham, 2017; Wilkins & Norton, 2018). The results of this study are in line with the theory which reveals that the meaning of fractions as part of a whole is a necessary condition for students to learn fractions (Charalambous & Pitta-Pantazi, 2007).

Furthermore, Table 2 shows that students indicated that they were experiencing learning difficulties because the meaning of the fractions expressed by students tended to be limited and some participants misinterpreted the meaning of the fractions. After conducting interviews with several participants, information was obtained that students with low EMS tended not to understand the meaning of the questions. If it is related to the theory related to the types of learning obstacles (Isnawan et al., 2022; Suryadi, 2019b), it is indicated that students experience learning obstacles with the type of ontogenic obstacle which is instrumental in nature. The results of this study are in line with (Fauzi & Suryadi, 2020a) which revealed that students indicated that they experienced learning difficulties with an ontogenic obstacle type that was instrumental because students did not understand the meaning of the questions.

In addition, based on information obtained from the mathematics teacher during interviews, students also indicated learning difficulties with the type of didactical obstacle from the point of view of didactic situation theory (Brousseau, 2002; Suryadi, 2019b). This is because math teachers cannot interpret fractions correctly. Mathematics teachers tend to interpret fractions as tools. Fractions are defined as tools that can make it easier for someone to share something. The results of this study are in line with (Tobias, 2013) which revealed that students tend to experience problems in learning fractions because the teacher's knowledge of mathematics is still lacking.

Moreover, students are also indicated to experience learning obstacles with the type of epistemological obstacle when viewed from theories related to types of learning obstacles (Brousseau, 2002; Suryadi, 2019b). Based on the results of interviews with several students and mathematics teachers, information was obtained that mathematics teachers rarely use illustrative models in learning fractions. In addition, the context used by mathematics teachers is limited or not varied so that students have difficulty implementing the concept of fractions in different contexts. Snippets of the researchers' interviews with mathematics teachers can be seen in Table 6. The results of this study are in line with several previous studies which revealed that mathematics teachers tend to rarely use illustrative models and variations in problem contexts in learning fractions at school (Mohamed et al., 2021; Thurlings et al., 2019).

Table 6
Snippet of math teacher interview answers

Researcher Questions	Math Teacher Answers
Have you ever started learning by giving students problems at the beginning?	*I have, but on average they are confused. They do not understand how to do it, how to do this, so let's explain it again first, this is how you do it, like this like this.*
Have not you tried using another model? For example, a number line.	*Number lines, yes, but they get confused. Yes, it's a bit difficult to use a number line.*

Based on Figure 1, information is obtained that the most dominant theme is students' lack of understanding of mathematical material. This study's results align with research (Özüdoğru, 2021), which revealed that students tend to experience problems with their low understanding of the content of learning materials during panic-gogy. Other research also revealed that students indicated decreased levels of understanding and interest in learning during panic-gogy (Frolova et al., 2021). Some factors that impact these phenomena are as follows: First, students and teachers lack skills in applying to learn using digital learning platforms; smartphone, quota, and internet signal limitations; and schools do not have an e-learning system that can be used during panic-gogy (Akar & Erden, 2021; Mailizar et al., 2020). This factor is then related to the third theme in Figure 1. The existence of the internet in the panic-gogy context has a reasonably dominant role because, when internet access is available, it is very likely that mathematics learning activities can be carried out (Zhou et al., 2020). Although there is no guarantee that it will run optimally.

Furthermore, the first theme is also related to the fourth theme. One of the factors that causes students to not understand math material during panic-gogy is that they tend to feel bored while studying at home, lack concentration during online learning, rely solely on Google for learning, and have parents. Students do not understand mathematics, so they do not have discussion partners when experiencing problems in learning. The results of this study are in line with several previous studies, which revealed that students tend to be unfocused, get bored quickly, and are less interested in learning during panic-gogy (Özüdoğru, 2021; Wijnker et al., 2021). This theme is also related to the seventh theme, which reveals that parents of students do not understand math material, so they have difficulty giving explanations when children ask about math material. Several previous studies have revealed that students perceive their parents as non-mathematics teachers and, therefore, cannot help during panic-gogy (Akar & Erden, 2021; Kalogeropoulos et al., 2021).

According to parents, the next problem quite dominant among students is that the teacher's explanation during panic-gogy in learning mathematics is lacking. Parents assume that students need a more detailed explanation from the teacher regarding the content of the material being studied and how to solve the problems presented by the teacher. These results are then in line with several previous studies, which revealed that students did not get optimal help from the teacher during panic-gogy in learning mathematics (Barlovits et al., 2021; Kalogeropoulos et al., 2021). In addition, parents also consider that the existence of panic disorder itself is a problem for students. In other words, the panic-gogy factor itself is to blame for the issues students encounter during panic-gogy. This study's results align with several theories and previous research, which revealed that mathematics teachers consider learning that is entirely carried out online to be a problem for students. The implementation of online learning can only be optimally utilized by mathematics teachers. The relationship that occurs between mathematics teachers and students tends to be one-way (Chirinda et al., 2021; Engelbrecht et al., 2020).

The fifth theme also links with other themes, especially the limited interaction between teachers and students during panic-gogy, which is quite dominant. In other words, the teacher's feedback to students during the panic-gogy of learning mathematics has not yet operated at its best. The results of this study are in line with several previous studies, which revealed that there were limitations to the communication process between students and teachers during panic-gogy and that the feedback given by teachers to students tended to be less than optimal (Demir & Demir, 2021; Özüdoğru, 2021). The characteristics of mathematics and math problems become the final theme identified in this study. Several theories also support the results of this study by revealing that the characteristics of mathematics, which are identical to formulas and arithmetic operations, are classic problems experienced by society in general (Clements & Sarama, 2018). Therefore, some experts suggest using various problems of everyday life as a starting point in learning and paying attention to the phases of the world of mathematics when studying at school (Tall, 2008).

Based on Table 4, information was obtained that most students needed to remember how to find the LCM value. Because there were no correct student answers, the researcher directed students to find the appropriate LCM values. After students seemed to understand how to find the LCM value, the researcher continued the learning activity by asking students what the GCD score was out of 4 and 6. In this activity, students also needed help determining the optimal GCD value. The researcher then explained by stating that the exact integers in each factor tree were the GCD values of the two numbers. This study's results align with the theory and research results, which reveal that students tend to have difficulty following learning activities at the beginning of meetings (Lodge et al., 2018; Tian & Siegler, 2017).

In connection with the *Let's Guess* Illustration Model activity, students can pair the illustration model with the appropriate integer. However, when students were asked about the reasons, they seemed unable to give a good explanation. As a result, the researcher offers justification in relation to the students' responses. This result is in line with the theory, which reveals that students tend to have difficulty giving reasons related to the answers given (Lin, 2018). The researcher continued learning through *Let's Read* activities. One student with ZA's initials volunteered to read a motivational story presented in a hybrid didactic design. After ZA finished reading, students with the initials SH and RAF conveyed the story's gist. SH stated that fractions are often used in commerce and agriculture. Meanwhile, the RAF stated that quite

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a lot of mathematical material is used in everyday life. The student's answer is in line with the theory, which reveals that fractions are pretty helpful for life (Obersteiner & Tumpek, 2016; Tian & Siegler, 2017).

The next activity the students did was *Let's Find Out*. In the breakout room (BOR), students worked in groups to complete this activity. Before entering the BOR, the researcher explained to the students how to do the *Let's Find Out* activity. The researcher described that students were allowed to answer their own problems according to their abilities (alternative method 1). The researcher also explained that students could use other alternative methods with the help of several illustrative models so that problems could be solved. This activity is intended so students' potential abilities can develop optimally (Topciu & Myftiu, 2015).

When the discussion activities began, students in group 2 needed help determining which students acted as note-takers. This study's results align with previous research, which revealed that students tend to have difficulty when discussing groups (Sofroniou & Poutos, 2016). The researcher then appointed the RAF as the minutes to resume the discussion activities. For group 1, the researcher directed students to actively discuss with other group members. Whereas for group 3, the researcher gave a trigger question by directing the student group to divide the illustration model into several equal parts. The researcher then returned to group 2 and directed students to divide the model illustrations according to the group's agreement. In addition, the researcher also directed students not to push themselves if they could not solve the problem independently but to use other alternative methods. Finally, group 2 uses the alternative method 1. One of the students was seen leading the discussion activity.

Unlike the previous group, group 1 was able to find the final answer. Researchers' direct students to determine students who will present the results obtained—only after the final minutes of the *Let's Find Out* activity group 3 had finished their work. The researcher then directed the student group to use the second alternative to solve the problem on time. The researcher returned all the students to the main room a few minutes later. This study's results align with previous research, which revealed that when learning, it is usually found that there are students or groups of students who tend to be slow when working on problems (Sofroniou & Poutos, 2016; Wester, 2020). The results of this study are also in line with the theory, which reveals that fractions can also mean the whole of the whole in the context of integers written in the form of fractions (Bennett et al., 2016).

In contrast to problem 1, none of the students could solve problem 2. Therefore, the researchers took the initiative together with the students to solve the problem. The results of this study are then in line with previous research, which revealed that not all problems in learning mathematics can be solved properly by students (Özreçberoğlu & Çağanağa, 2018). The results of this study are also in line with theory and research, which reveal that the use of illustration or image models tends to be liked by students in learning (Ervin, 2017; Mamonto et al., 2018). Students also emphasized that the meaning of fractions found at this meeting was fractions as part of the whole and fractions as ratios or comparisons. In addition, students revealed material they had yet to understand.

When compared with theories related to the meaning of fractions (Bennett et al., 2016), from Table 5, information is obtained that students with low EMS have been able to express the three meanings of appropriate fractions, namely fractions as ratios, sizes, and operators. For students with moderate EMS, information was obtained that students were able to express the five meanings of fractions that should be. Likewise, for students with high EMS, Students should have expressed all the meanings of fractions. The results of this study are in line with (Getenet & Callingham, 2017), which revealed that students could interpret fractions as a whole.

Table 5 also provides information that fractions as ratios become the dominant meaning of fractions after implementing a hybrid didactic design. The results of this study are in line with (Mamonto et al., 2018), which revealed that students tend to interpret fractions as a comparison between the numerator and denominator. In contrast to previous research, several studies have revealed that students tend to interpret fractions as part of a whole, size, and operators (Charalambous & Pitta-Pantazi, 2007). These different results are due to the characteristics of the participants in the study, which tend to be different. Students at the junior high school level tend to interpret fractions as ratios, while students at the elementary level tend to interpret fractions as part of a whole. This result is because students' thinking skills at the junior high school level tend to be axiomatic-formal (Isnawan et al., 2022; Tall, 2008).

Therefore, the conclusion for this section is that there are indications that students do not experience learning difficulties when interpreting fractions. Students experience limitations in interpreting fractions, not because of external factors, such as hybrid didactic designs, but because of the students' internal factors (Isnawan et al., 2022). Additionally, a student with high EMS experiences exciting things. These students tend to need to be more consistent when working on them. The answers that students convey during the exam are fractions as a whole. However, when interviewed, students revealed that the intended meaning was fractions as part of the whole. When compared with theories related to types of learning obstacles (Suryadi, 2019b), it can be concluded that students experience learning obstacles with psychological ontogenic obstacles. This study's results align with several previous studies, which revealed that students tend to experience learning obstacles with psychological ontogenic obstacles when interpreting fractions (Fauzi & Suryadi, 2020b, 2020a).

The results of this study are in line with previous research, which revealed that didactic designs were able to get positive responses from students (Isnawan et al., 2022; Ruli et al., 2019). This study's results align with research that reveals that students must be given space to convey ideas related to what they have read in learning mathematics (Lin, 2018). The results of this study are then in line with the theory, which reveals that there is no single learning design that, when implemented, can get a response according to what the teacher predicts (Suryadi, 2019b, 2019a). In other words, there will definitely be changes or revisions to the design after the implementation of learning activities.

The findings in this study provide an overview that there are several significant differences in obstacles between distance learning in mathematics and panic-gogy conditions. The first obstacle is that the level of student boredom in learning is higher during panic-gogy. This is because, during panic-gogy, all learning is forced to be done online, not due to the wishes of students or teachers. In contrast, distance learning conditions are usually the choice of students or teachers, so students' psychological conditions are much more ready for online learning.

The second obstacle is that the learning mode changes rapidly during panic-gogy, requiring teachers to prepare teaching materials that can be used for all types of modes. Meanwhile, in distance learning, there is only one mode—online learning—so the teaching materials provided are also of one type: online teaching materials. For example, researchers in this study use a hybrid didactic design in learning fractions. The hybrid didactic design can be used in various learning modes, such as online, limited face-to-face, and home visit. The third obstacle is that students during panic-gogy need printed teaching materials (hybrid didactic design). This is because students during online learning in panic-gogy use smartphones, not laptops, making it difficult for them to share screens. Therefore, students choose to write their work on printed teaching materials and then photograph and share it via *WhatsApp* groups or show it directly to the smartphone camera.

The fourth obstacle is that the distribution of printed teaching materials is not easy during panic-gogy. In addition to restrictions on interaction, students' home addresses also tend to be difficult to find. Good coordination is needed between researchers and schools, such as guidance and counseling teachers and vice principals for curriculum, to locate students' addresses and schedule study sessions according to the current learning mode. The last obstacle is the limited access to devices or internet connections among students during panic-gogy. This differs from distance learning conditions, where participants usually already have the necessary devices before choosing the learning mode. The solution provided in this study is that students who do not have devices are asked to follow the home-visit learning mode, while students with limited internet access are given data quota assistance by the researcher.

Limitations

This research has limitations in several ways, such as not all students being able to participate in learning activities in online modes. Therefore, future research will look for alternative learning mode solutions suitable for students who cannot participate in online learning activities. Additionally, this research will require a large amount of money for the process of printing hybrid didactic designs, so the researchers suggest that in future research, a revised hybrid didactic design will be compiled in the form of a website or *Android*. This is intended to streamline printing costs, and all students or teachers will be able to access the module, both at school and outside of school. Apart from that, excessive time allocation is another limitation in this research, impacting the allocation of time for other learning. Therefore, future research includes group reflection activities or lesson study activities when compiling, implementing, and reflecting on didactic designs to obtain more efficient learning designs.

5. Conclusion

Based on the previous description, several findings can be concluded. Before implementation, fractions as rational numbers had been the dominant meaning of fractions expressed by students for all EMS. In other words, most students indicated that they had experienced learning obstacles because they could not interpret fractions fully. To minimize these obstacles, the researcher developed a hybrid didactic design that could be implemented during panic-gogy. The hybrid didactic design included at least a few activities, such as helping students understand the different meanings of fractions to get around didactical problems and using a range of illustrative models to get around epistemological problems that could get in the way of learning. The researcher used various triggering questions combined with illustrative models to overcome ontogenic instrumental obstacles. After implementing the hybrid didactic design, information was obtained that most participants could correctly express the meaning of the five fractions. However, it was found that there were participants who needed correction in expressing the meaning of fractions. These results indicated that students experienced learning obstacles. However, after the interviews were conducted, information was obtained that the participants needed more time to study, so students were not optimal in answering during exams. In other words, there were indications that participants did not experience learning obstacles.

Based on the previous description, a hybrid didactic design appears to be an alternative that mathematics teachers can use, particularly when teaching students the meaning of fractions. Moreover, when teachers want to minimize the learning obstacles that students experience, hybrid didactic designs also tend to be quite relevant for use by teachers in distance learning mode because they optimize student activities during learning. However, it should be noted that when a mathematics teacher decides to use distance learning, the distribution of the printed version of the didactic design must be obtained by students before online learning activities take place. This is intended to anticipate various obstacles during online learning, such as an unstable internet connection.

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Author Contribution
MGI : Conceptualization, Writing - Original Draft, Editing and Visualization;
NMA : Writing - Review & Editing, Formal analysis, and Methodology;
RDK : Validation and Supervision.

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The authors used ChatGPT (OpenAI) solely to enhance language clarity, improve grammar, and assist with proofreading during the manuscript preparation. The tool was not used to generate research ideas, analyze data, interpret findings, or create substantive academic content. All scientific arguments, methodological decisions, results, and conclusions are entirely the authors' own work

6. References

Adu-gyamfi, K., Schwartz, C. S., Sinicrope, R., & Bosse, M. J. (2019). Making sense of fraction division: Domain and representation knowledge of preservice elementary teachers on a fraction division task. *Mathematics Education Research Journal*, *31*, 502–528. https://doi.org/10.1007/s13394-019-00265-2

Akar, S. S., & Erden, M. K. (2021). Distance education experiences of secondary school math teacher during the pandemic: A narrative study. *Turkish Online Journal of Distance Education*, *22*(3), 1–20. https://files.eric.ed.gov/fulltext/EJ1301012.pdf

Barlovits, S., Jablonski, S., Ludwig, M., & Recio, T. (2021). Teaching from a distance-Math sessons during COVID-19 in Germany and Spain. *Education Sciences*, *11*(406), 1–17. https://doi.org/10.3390/educsci11080406

Bayaga, A., & Bossé, M. J. (2018). Semantic and syntactic fraction understanding. *International Electronic Journal of Elementary Education*, *11*(2), 135–142. https://doi.org/10.26822/iejee.2019248587

Bennett, A. B., Burton, L. J., Nelson, L. T., & Ediger, J. R. (2016). *Mathematics for elementary teachers: A conceptual approach*. McGraw-Hill Education.

Brousseau, G. (2002). *Theory of didactical situations in mathematics*. Kluwer Academic Publishers. https://id1lib.org/book/979725/fd6fae

Brousseau, G., & Warfield, V. (2020). Didactical Situations in Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (2nd ed., pp. 206–213). Springer.

Brown, A., & Danaher, P. A. (2017). CHE principles: Facilitating authentic and dialogical semi-structured interviews in educational research structured interviews in educational research. *International Journal of Research & Method in Education*, *42*(1), 76–90. https://doi.org/10.1080/1743727X.2017.1379987

Capinding, A. T. (2022). Impact of modular distance learning on high school students mathematics motivation, interest/attitude, anxiety and achievement during the Covid-19 pandemic. *European Journal of Educational Research*, *11*(2), 917–934. https://doi.org/10.12973/eu-jer.11.2.917

Charalambous, C. Y., & Pitta-Pantazi, D. (2007). Drawing on a theoretical model to study students' undertandings of fractions. *Educational Studies in Mathematics*, *64*, 293–316. https://doi.org/10.1007/s

Chirinda, B., Ndlovu, M., & Spangenberg, E. (2021). Teaching mathematics during the Covid-19 lockdown in a context of historical disadvantage. *Education Sciences*, *11*(117), 1–14. https://doi.org/10.3390/educsci11040177

Clements, D. H., & Sarama, J. (2018). Myths of early math. *Education Sciences*, *8*(71), 1–8. https://doi.org/10.3390/educsci8020071

Cortina, J. L., Visnovska, J., & Zuniga, C. (2014). Unit fractions in the context of proportionality: Supporting students' reasoning about the inverse order relationship. *Mathematics Education Research Journal*, *26*, 79–99. https://doi.org/10.1007/s13394-013-0112-5

Cramer, K., Monson, D., Ahrendt, S., Wyberg, T., Fagerlund, C., Cramer, K., Monson, D., Ahrendt, S., & Wyberg, T. (2018). Investigations in mathematics learning reconstructing the unit on the number line: Tasks to extend fourth graders' fraction understandings. *Investigations in Mathematics Learning*, 1–15. https://doi.org/10.1080/19477503.2018.1434594

Creswell, J. W., & Creswell, J. D. (2018). *Research design: Qualitative, quantitative, and mixed methods approaches* (H. Salmon, Ed.; 5th ed.). Sage Publication, Inc. https://id1lib.org/book/3700358/d95149

Dalkin, S., Forster, N., Hodgson, P., Lhussier, M., & Car, S. M. (2020). Using computer assisted qualitative data analysis software (CAQDAS; NVivo) to assist in the complex process of realist theory generation, refinement and testing. *International Journal of Social Research Methodology*, *24*(1), 123–134. https://doi.org/10.1080/13645579.2020.1803528

Demir, E., & Demir, C. G. (2021). Investigation of parents' opinions about distance education during the COVID-19 pandemic. *Turkish Online Journal of Distance Education*, *22*(2), 42–57. https://files.eric.ed.gov/fulltext/EJ1290801.pdf

Dewi, D., Suryadi, D., Suratno, T., Mulyana, E., & Kurniawan, H. (2016). Meaning of fractions. *Journal of Physics: Conference Series*, *755*(1), 1–6. https://doi.org/10.1088/1742-6596/755/1/011001

Engelbrecht, J., Llinares, S., & Borba, M. C. (2020). Transformation of the mathematics classroom with the internet. *ZDM Mathematics Education*, *52*(5), 825–841. https://doi.org/10.1007/s11858-020-01176-4

Ervin, H. K. (2017). Fraction multiplication and division models: A practitioner reference paper. *International Journal of Research in Education and Science*, *3*(1), 258–279. https://files.eric.ed.gov/fulltext/EJ1126700.pdf

Fauzi, I., & Suryadi, D. (2020a). Learning obstacle the addition and subtraction of fraction in grade 5 elementary schools. *MUDARRISA: Jurnal Kajian Pendidikan Islam*, *12*(1), 51–68. https://doi.org/10.18326/mdr.v12i1.51-68

Fauzi, I., & Suryadi, D. (2020b). The Analysis of Students' Learning Obstacles on the Fraction Addition Material for Five Graders of Elementary Schools. *Al Ibtida: Jurnal Pendidikan Guru MI*, *7*(1), 33–45. https://doi.org/10.24235/al.ibtida.snj.v7i1.6020

Frolova, E. V, Rogach, O. V, Tyurikov, A. G., & Razov, P. V. (2021). Online student education in a Pandemic: New challenges and risks. *European Journal of Contemporary Education*, *10*(1), 43–52. https://doi.org/10.13187/ejced.2021.1.43

Getenet, S., & Callingham, R. (2017). Teaching fractions for understanding: addressing interrelated concepts. *In: 40th Annual Conference of the Mathematics Education Research Group of Australasia: 40 Years On: We Are Still Learning! (MERGA40)*, *1983*, 277–284. https://files.eric.ed.gov/fulltext/ED589552.pdf

Getenet, S., & Callingham, R. (2021). Teaching interrelated concepts of fraction for understanding and teacher's pedagogical content knowledge. *Mathematics Education Research Journal*, *33*, 201–221. https://doi.org/10.1007/s13394-019-00275-0

Gómez-Chacón, I. M. (2017). Emotions and heuristics: The state of perplexity in mathematics. *ZDM-Mathematics Education*, *49*(3), 323–338. https://doi.org/10.1007/s11858-017-0854-8

Harel, G. (2008). What is mathematics? A pedagogical answer to a philosophical question. In B. Gold & R. A. Simons (Eds.), *Proof and other dilemmas: Mathematics and philosophy* (pp. 265–290). The Mathematical Assosiation of America. https://www.ams.org/books/spec/059/

Harvey, R. (2012). Stretching student teachers' understanding of fractions. *Mathematics Education Research Journal*, *24*, 493–511. https://doi.org/10.1007/s13394-012-0050-7

Isnawan, M. G. (2022). *Desain didaktis pembelajaran pecahan di SMP Negeri 1 Narmada Kabupaten Lombok Barat* [Didactic design of fraction learning at SMP Negeri 1 Narmada, West Lombok Regency][Dissertation]. Universitas Pendidikan Indonesia.

Isnawan, M. G., Suryadi, D., & Turmudi, T. (2022). How do secondary students develop the meaning of fractions? A hermeneutic phenomenological study. *Beta: Jurnal*, *15*(1), 1–19. https://doi.org/10.20414/betajtm.v15i1.496

Isnawan, M. G., & Alsulami, N. M. (2024). Didactical design for online learning in ordering fractions. *International Journal of Didactic Mathematics in Distance Education*, *1*(1), 1–12. https://doi.org/10.33830/ijdmde.v1i1.7653

Job, P., & Schneider, M. (2014). Empirical positivism, an epistemological obstacle in the learning of calculus. *ZDM - International Journal on Mathematics Education*, *46*(4), 635–646. https://doi.org/10.1007/s11858-014-0604-0

Kalogeropoulos, P., Roche, A., Russo, J., Vats, S., & Russo, T. (2021). Learning mathematics from home during COVID-19: insights from two inquiry-focussed primary schools. *Eurasia Journal of Mathematics, Science and Technology Education*, *17*(5), 1–16. https://doi.org/10.29333/ejmste/10830

Kamanetz, A. (2020). *'Panic-gogy': Teaching online classes during the coronavirus pandemic*. Npr. https://www.npr.org/2020/03/19/817885991/panic-gogy-teaching-online-classes-during-the-coronavirus-pandemic

Kor, L.-K., Teoh, S.-H., Mohamed, S. S. E. B., & Singh, P. (2018). Learning to make sense of fractions: Some insights from the Malaysian primary 4 pupils. *International Electronic Journal of Mathematics Education*, *14*(1), 169–182. https://doi.org/10.29333/iejme/3985

Lawshe, C. H. (1975). A quantitative approach to content validity. *Personel Psychology*, *28*, 563–575. https://doi.org/10.1111/J.1744-6570.1975.TB01393.X

Lin, P. J. (2018). The development of students mathematical argumentation in a primary classroom. *Educacao and Realidade*, *43*(3), 1171–1192. https://doi.org/10.1590/2175-623676887

Lodge, J. M., Kennedy, G., Lockyer, L., Arguel, A., & Pachman, M. (2018). Understanding difficulties and resulting confusion in learning: An integrative review. *Frontiers in Education*, *3*. https://doi.org/10.3389/feduc.2018.00049

Mailizar, M., Almanthari, A., Maulina, S., & Bruce, S. (2020). Secondary school mathematics teachers' views on e-learning implementation barriers during the COVID-19 pandemic: The case of Indonesia. *Eurasia Journal of Mathematics, Science and Technology Education*, *16*(7). https://doi.org/10.29333/EJMSTE/8240

Mamonto, K., Juniati, D., & Siswono, T. Y. E. (2018). Understanding fraction concepts of Indonesian junior high school students: A case of field independent and field dependent students. *Journal of Physics: Conference Series*, *947*, 1–7. https://doi.org/10.1088/1742-6596/947/1/012058

Marfuah, M., Suryadi, D., Turmudi, T., & Isnawan, M. G. (2022). Providing online learning situations for in-service mathematics teachers' external transposition knowledge during Covid-19 pandemic: Case of Indonesia. *Electronic Journal of E-Learning*, *20*(1), 69–84. https://doi.org/10.34190/ejel.20.1.2388

Martinez, S., & Blanco, V. (2021). Analysis of problem posing using different fractions meanings. *Education Sciences*, *11*(65), 1–12. https://doi.org/10.3390/educsci11020065

Miles, M. B., Huberman, A. M., & Saldana, J. (2014). Qualitative data analysis: A methods sourcebook. In 3rd ed. SAGE Publications, Inc. https://id.id1lib.org/book/3593988/83e08f

Mohamed, R., Ghazali, M., & Samsudin, M. A. (2021). A systematic review on teaching fraction for understanding through representation on Web of Science database using PRISMA. *LUMAT: International Journal on Math, Science and Technology Education*, *9*(1), 100–125. https://doi.org/10.31129/LUMAT.9.1.1449

Nowell, L. S., Norris, J. M., White, D. E., & Moules, N. J. (2017). Thematic analysis: Striving to meet the trustworthiness criteria. *International Journal of Qualitative Methods*, *16*, 1–13. https://doi.org/10.1177/1609406917733847

Obersteiner, A., & Tumpek, C. (2016). Measuring fraction comparison strategies with eye-tracking. *ZDM*, *48*, 255–266. https://doi.org/10.1007/s11858-015-0742-z

Özreçberoğlu, N., & Çağanağa, Ç. K. (2018). Making it count: strategies for improving problem-solving skills in mathematics for students and teachers' classroom management. *Eurasia Journal of Mathematics, Science and Technology Education*, *14*(4), 1253–1261. https://doi.org/10.29333/ejmste/82536

Özüdoğru, G. (2021). Problems faced in distance education during Covid-19 Pandemic. *Participatory Educational Research*, *8*(4), 321–333. https://doi.org/10.17275/per.21.92.8.4

Prabowo, A., Suryadi, D., Dasari, D., Juandi, D., & Junaedi, I. (2022). Learning obstacles in the making of lesson plans by prospective mathematics teacher students. *Education Research International*, *2022*, 1–15. https://doi.org/10.1155/2022/2896860

Prasetya, P. L., & Mahmudah, F. N. (2021). Mathematics learning using zoom cloud meeting during the Covid-19 pandemic for elementary school students. *Pedagogik Journal of Islamic Elementary School*, *4*(1), 45. https://doi.org/10.24256/pijies.v4i1.1776

Prihandhika, A., Suryadi, D., & Prabawanto, S. (2022). The investigation of concept image towards derivative representation: A case study of prospective mathematics teachers. *Mathematics Teaching Research Journal*, *14*(4), 148–164. https://commons.hostos.cuny.edu/mtrj/wp-content/uploads/sites/30/2022/12/08-Aditya.pdf

Prihandhika, A., & Perbowo, K. S. (2024). The review of concept image and concept definition: A hermeneutic phenomenological study on the derivative concepts. *International Journal of Didactic Mathematics in Distance Education*, *1*(1), 13–23. https://doi.org/10.33830/ijdmde.v1i1.7610

Ruli, R. M., Prabawanto, S., & Mulyana, E. (2019). Didactical design research of quadratic function based on learning obstacle and learning trajectory. *Journal of Physics: Conference Series*, *1157*, 1–6. https://doi.org/10.1088/1742-6596/1157/4/042060

Santagata, R., König, J., Scheiner, T., Nguyen, H., Adleff, A. K., Yang, X., & Kaiser, G. (2021). Mathematics teacher learning to notice: A systematic review of studies of video-based programs. *ZDM-Mathematics Education*, *53*(1), 119–134. https://doi.org/10.1007/s11858-020-01216-z

Sasidharan, S., & Kareem, J. (2023). Student perceptions and experiences in mathematics classrooms: A thematic analysis. *International Journal of Innovation in Science and Mathematics Education*, *31*(2), 47–59. https://doi.org/10.30722/IJISME.31.02.004

Siagian, M. D., Suryadi, D., Nurlaelah, E., & Prabawanto, S. (2022). Investigation of secondary students' epistemological obstacles in the inequality concept. *Mathematics Teaching Research Journal*, *14*(4), 106–128. https://commons.hostos.cuny.edu/mtrj/wp-content/uploads/sites/30/2022/12/06-Siagian.pdf

Simon, M. A., Placa, N., Avitzur, A., & Kara, M. (2018). Promoting a concept of fraction-as-measure: A study of the Learning Through Activity research program. *Journal of Mathematical Behavior*, *52*, 122–133. https://doi.org/10.1016/j.jmathb.2018.03.004

Sofroniou, A., & Poutos, K. (2016). Investigating the effectiveness of group work in mathematics. *Education Sciences*, *6*(3), 13-28. https://doi.org/10.3390/educsci6030030

Sopacua, J., Fadli, M. R., & Rochmat, S. (2020). The history learning module integrated character values. *Journal of Education and Learning*, *14*(3), 463–472. https://doi.org/10.11591/edulearn.v14i3.16139

Sukarma, I. K., Isnawan, M. G., & Alsulami, N. M. (2024). Research on nonroutine problems: A hybrid didactical design for overcoming student learning obstacles. *Human Behavior and Emerging Technologies*, *2024*(Article ID 5552365), 1–12. https://doi.org/10.1155/2024/5552365

Sun, X. H. (2019). Bridging whole numbers and fractions: Problem variations in Chinese mathematics textbook examples. *ZDM*, *51*, 109–123. https://doi.org/10.1007/s11858-018-01013-9

Suryadi, D. (2019a). *Landasan filosofis penelitian desain didaktis (DDR)*. Pusat Pengembangan DDR Indonesia.

Suryadi, D. (2019b). *Penelitian desain didaktis (DDR) dan implementasinya*. Gapura Press.

Tall, D. (2008). The transition to formal thinking in mathematics. *Mathematics Education Research Journal*, *20*(2), 5–24. https://doi.org/10.1007/BF03217474

Thurlings, M., Koopman, M., den Brok, P., & Pepin, B. (2019). Portraying primary fraction teaching: A variety of mathematical richness, pedagogic strategies, and use of curriculum materials. *International Journal of Education in Mathematics, Science and Technology*, *7*(2), 170–185. https://doi.org/10.18404/ijemst.552452

Tian, J., & Siegler, R. S. (2017). Fractions learning in children with mathematics difficulties. *Journal of Learning Disabilities*, *50*(6), 614–620. https://doi.org/http://dx.doi.org/10.1177/0022219416662032

Tobias, J. M. (2013). Prospective Elementary Teachers' Development of Fraction Language for Defining the Whole. *Journal of Mathematics Teacher Education*, *16*(2), 85–103. https://doi.org/10.1007/s10857-012-9212-5

Topciu, M., & Myftiu, J. (2015). Vygotsky theory on social interaction and its influence on the development of pre-school. *European Journal of Social Science Education and Research*, *2*(3), 172–179. https://doi.org/10.26417/ejser.v4i1.p172-179

Veldman, D. J., & Sanford, J. P. (1984). The influence of class ability level on student achievement and classroom behavior. *American Educational Research Journal*, *21*(3), 629–644. https://doi.org/10.3102/00028312021003629

Veloo, P. K., & Puteh, M. (2017). Mathematical knowledge in teaching fraction concepts. *Oral Science International*, *29*(2), 109–114. http://www.sci-int.com/pdf/636296608941203198.pdf

Villalba, M. del C. M., Salgado, A. M., & Marin, Y. M. (2019). Addition between fractions as part of a whole using play with a game of A3 strips. *Panorama*, *13*(25). https://doi.org/10.15765/pnrm.v13i25.1265

Wahyu, K. (2021). How students understand smaller fractions divided by greater fractions? *Beta: Jurnal Tadris Matematika*, *14*(1), 85–92. https://doi.org/10.20414/betajtm.v14i1.447

Wahyu, K., Kuzu, T. E., Subarinah, S., Ratnasari, D., & Mahfudy, S. (2020). Partitive fraction division: Revealing and promoting primary students' understanding. *Journal on Mathematics Education*, *11*(2), 237–258. https://doi.org/http://doi.org/10.22342/jme.11.2.11062.237-258

Wester, J. S. (2020). Students' possibilities to learn from group discussions integrated in whole-class teaching in mathematics. *Scandinavian Journal of Educational Research*, 1–17. https://doi.org/10.1080/00313831.2020.1788148

Wijnker, W., Bakker, A., Schukajlow, S., & Drijvers, P. (2021). Modeling the mechanisms of interest raising videos in education. *Human Behavior and Emerging Technologies*, *3*(5), 823–831. https://doi.org/10.1002/hbe2.314

Wilkins, J. L. M., & Norton, A. (2018). Learning progression toward a measurement concept of fractions. *International Journal of STEM Education RESEARCH*, *5*(27), 1–11. https://doi.org/10.1186/s40594-018-0119-2

Yerushalmy, M., & Olsher, S. (2020). Online assessment of students' reasoning when solving example-eliciting tasks: Using conjunction and disjunction to increase the power of examples. *ZDM-Mathematics Education*, *52*(5), 1033–1049. https://doi.org/10.1007/s11858-020-01134-0

Zhang, X., Clements, M. A. K., & Ellerton, N. F. (2014). Conceptual mis(understandings) of fractions: From area models to multiple embodiments. *Mathematics Education Research Journal*, *27*(2), 233–261. https://doi.org/10.1007/s13394-014-0133-8

Zhou, L., Li, F., Wu, S., & Zhou, M. (2020). "School's out, but class's on", the largest online education in the world today: Taking China's practical exploration during the COVID-19 Epidemic prevention and control as an example. *Best Evidence in Chinese Education*, *4*(2), 501–519. https://doi.org/10.15354/bece.20.ar023