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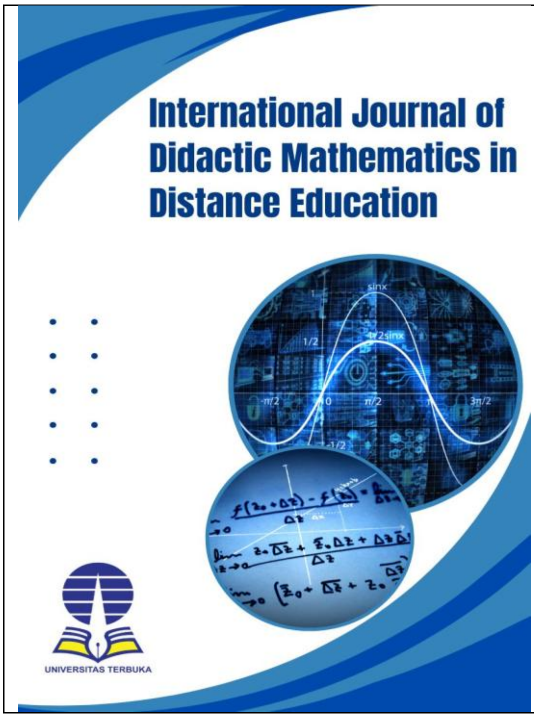

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Reconceptualizing authenticity in problem-based learning for linear algebra course: A multi-stakeholder analysis of video-based hybrid instruction

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Abstract
This study develops an integrated video-based hybrid online learning model with problem-based learning (PBL) for Linear Algebra instruction in resource-constrained contexts. A qualitative case study with three experts and five students using thematic analysis yielded 13 codes organized into five themes: technological challenges, video characteristics, hybrid architecture, PBL implementation, and motivation strategies. Findings reveal that 100% of participants identified infrastructure constraints as primary barriers, necessitating downloadable mobile-optimized content. Optimal video specifications include 15-20 minute instructor-led explanations with embedded interactivity. The study reconceptualizes 'authentic problems' for abstract mathematics from contextual realism to conceptual genuineness, demonstrating that effective PBL engages genuine mathematical thinking—comparing representations, investigating theorems, exploring counterexamples, rather than forced real-world applications. The developed three-phase hybrid model integrates asynchronous video foundation, synchronous collaborative problem-solving, and asynchronous portfolio consolidation. Multi-stakeholder convergence on core principles provides robust design guidance, while divergences regarding scaffolding and accessibility illuminate implementation challenges. The study contributes theoretically by challenging traditional PBL frameworks for abstract domains and methodologically by demonstrating that integrating expert pedagogical knowledge with student experiential perspectives yields contextually feasible models. Practical implications address institutional digital learning policies, LMS developer feature priorities, and educator implementation guidelines adaptable across resource-constrained contexts globally.

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1. Introduction

Digital technologies have fundamentally transformed the scope and delivery modalities of higher education, with the COVID-19 pandemic serving as an unprecedented catalyst for rapid adoption of online and hybrid learning models across educational institutions worldwide (Timotheou et al., 2023). The integration of video-based learning and hybrid instructional frameworks has emerged as a critical strategy for addressing pedagogical challenges in abstract subjects, driven simultaneously by demands for flexible and accessible educational delivery and by recognition that emerging technologies can potentially enhance learning outcomes when pedagogically grounded (Hashim et al., 2022). However, mathematics-based courses, particularly those emphasizing abstract theoretical content such as Linear Algebra, present distinctive challenges that transcend general online learning concerns, necessitating specialized approaches that strategically combine technological innovation with pedagogical methods specifically designed to support the deep conceptual understanding that abstract mathematics requires (Stewart et al., 2019; Harel, 2017).

Within this broader landscape of digital transformation in higher education, mathematics education, particularly abstract mathematical domains, faces unique implementation challenges requiring specialized pedagogical consideration. Linear Algebra presents particularly acute pedagogical challenges due to its highly abstract nature, with students consistently struggling to comprehend vector space concepts, linear transformations, eigenvalues, and other theoretical structures that require sophisticated spatial reasoning and symbolic manipulation (Stewart et al., 2019; Harel, 2017). This difficulty is substantially compounded in online learning environments, where the absence of face-to-face interaction limits instructors' ability to gauge student comprehension through visual cues and provide immediate targeted clarification when misconceptions emerge (Harel, 2017). Meta-analytic evidence reveals that web-based algebra instruction demonstrates only moderate effectiveness ($g = 0.73$, $p < .01$), substantially lower than other mathematical domains such as statistics and probability ($g = 2.86$), suggesting that abstract mathematical content requires specialized pedagogical approaches beyond standard online delivery methods (Li et al., 2022). As a foundational course for science, technology, engineering, economics, and mathematics disciplines, enhancing Linear Algebra instruction effectiveness has implications extending far beyond mathematics education itself, potentially influencing student success and retention across multiple STEM pathways.

PBL has emerged as a promising pedagogical approach for mathematics instruction, grounded in constructivist principles that emphasize active knowledge construction through engagement with challenging problems. Emerging evidence demonstrates PBL's effectiveness across diverse mathematical contexts: problem-based digital learning significantly enhances both mathematics proficiency ($F(1,58) = 12.47$, $p < .01$) and creative problem-solving among vocational students (Chen et al., 2025), while producing learning outcomes equivalent to or exceeding conventional instruction in secondary mathematics (Abdullah et al., 2010). However, systematic reviews reveal a critical gap, research on technology-supported PBL for abstract mathematics in higher education, particularly for advanced courses like Linear Algebra, remains severely limited (Abdullah et al., 2024). This gap is particularly consequential given PBL's theoretical alignment with constructivist epistemology, which posits that conceptual understanding of abstract structures must be actively constructed rather than passively received, precisely the cognitive processes required for developing robust understanding in Linear Algebra (Hmelo-Silver, 2004). Nevertheless, how PBL can be effectively adapted for abstract mathematical content in technology-mediated environments remains an open empirical question requiring systematic investigation.

Complementing problem-based pedagogical approaches, video-based learning technologies offer distinct affordances that may address some inherent challenges of online mathematics instruction. Video-based learning enables students to control learning pace, replay complex explanations multiple times, and access content asynchronously despite connectivity constraints, capabilities particularly valuable for processing abstract mathematical concepts that require extended cognitive engagement (Kong & Li, 2020). Research in mathematics education demonstrates that carefully designed instructional videos significantly improve mathematical achievement and cultivate more positive student attitudes toward challenging content (Kong & Li, 2020). Analysis of learning resource usage patterns reveals that students strategically combine live lectures and asynchronous video resources in ways that optimize their individual learning needs and preferences, suggesting that hybrid models leveraging both modalities may be more effective than either approach alone (O'Dwyer et al., 2018). Recent framework development for instructional videos in mathematical modeling has identified specific design principles, including optimal duration, visual format, pacing, and interactive elements, that enhance effectiveness, though students simultaneously report both advantages and persistent challenges in video-based learning contexts (Rach & Ufer, 2024). Nevertheless, research specifically examining video learning integration with problem-based approaches in hybrid contexts for Linear Algebra courses remains conspicuously absent from the literature.

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While these individual lines of research demonstrate promise, their isolated development has created critical knowledge gaps at the intersection of video learning, hybrid environments, and problem-based pedagogy for abstract mathematics instruction. Despite growing bodies of literature on hybrid learning, PBL, and video-based instruction pursued independently, critical gaps remain at their nexus. Existing research has not examined how video-based hybrid learning environments can be specifically designed to support problem-based approaches for abstract mathematical content like Linear Algebra, where the nature of concepts differs substantially from applied or computational mathematics emphasized in most educational technology research. Moreover, prior studies typically adopt single-stakeholder perspectives, focusing either on instructor experiences or student outcomes, thereby overlooking the multi-dimensional insights necessary for developing pedagogically sound and practically implementable models (Fullan, 2016). Perhaps most significantly, literature addressing technological affordances (e.g., video characteristics, platform capabilities) and pedagogical strategies (e.g., PBL problem design, collaboration structures) tends to treat these elements in isolation rather than investigating their potential synergies for enhancing conceptual understanding in abstract mathematical domains (Picciano, 2009). This fragmentation limits both theoretical understanding of how multimedia learning principles interact with constructivist pedagogy and practical guidance for educators seeking to optimize mathematics learning in increasingly hybrid educational contexts, particularly in resource-constrained environments where technological limitations necessitate creative pedagogical adaptations.

Addressing these interconnected research gaps carries both theoretical and practical significance for mathematics education and educational technology fields. Theoretically, this study contributes to the literature by examining the intersection of multimedia learning principles (Mayer, 2021), constructivist pedagogy (Schmidt et al., 2011), and hybrid instructional design (Garrison & Kanuka, 2004) within the specific context of abstract mathematical concept learning, a domain where cognitive demands, knowledge structures, and learning processes differ substantially from the applied or computational mathematics contexts that dominate existing research. By investigating how video design characteristics and PBL implementation strategies can be synergistically combined to support abstract conceptual understanding, the study advances theoretical frameworks for technology-enhanced mathematics education. Practically, the study offers evidence-based, actionable guidance for mathematics educators, instructional designers, and educational technology developers seeking to optimize online and hybrid learning experiences in contexts characterized by diverse student populations and variable technological infrastructure. By grounding model development in multi-stakeholder perspectives, integrating expert pedagogical knowledge with student learning experiences, the study ensures that recommendations are simultaneously pedagogically rigorous, contextually appropriate, and implementable within real-world institutional constraints faced by higher education institutions globally, particularly those in developing contexts where resource limitations require innovative solutions.

Based on the identified research gaps and their significance, this study addresses the central research question: What are the essential design principles, implementation specifications, and pedagogical strategies for video-based hybrid online learning models integrating PBL approaches for Linear Algebra instruction, as identified through systematic analysis of expert pedagogical knowledge and student learning experiences? To answer this question comprehensively and generate actionable findings, the study pursues three specific research objectives: (1) to identify and critically analyze expert and student perspectives on the pedagogical needs, technological constraints, learning challenges, and support requirements specific to Linear Algebra instruction in hybrid online contexts; (2) to determine optimal video learning characteristics (including duration, visual format, content structure, interactivity features, and accessibility considerations) and PBL implementation strategies (including problem types, conceptual focus, collaboration structures, and scaffolding approaches) that are specifically suited to abstract mathematical content; and (3) to synthesize multi-stakeholder insights into a theoretically grounded, empirically informed, and contextually feasible video-based hybrid learning model with integrated PBL approaches, specifying operational components, implementation phases, design principles, and quality criteria for Linear Algebra courses and potentially other abstract mathematics disciplines.

2. Method

2.1. Research Design

This study employed a qualitative approach with an exploratory case study design to investigate video-based hybrid online learning models with PBL approaches for Linear Algebra courses from multi-stakeholder perspectives. The qualitative paradigm was selected over quantitative or mixed-methods approaches because the study's primary aim, developing a theoretically grounded instructional model by understanding stakeholder experiences, needs, and perspectives, required rich descriptive data that captures the complexity and nuance of human experiences in emerging educational contexts (Creswell, 2018). Qualitative inquiry enables researchers to explore "how" and "why" questions, investigate phenomena within their natural contexts, and generate in-depth understanding of complex social processes (Yin, 2018), all of which aligned with this study's objectives.

The specific choice of case study design was justified by several methodological considerations. Case study methodology is particularly appropriate when the research focuses on contemporary phenomena within real-life contexts, the boundaries between phenomenon and context are not clearly evident, and the researcher seeks to develop detailed, holistic understanding (Yin, 2018). This study examined the contemporary phenomenon of hybrid online learning with PBL for Linear Algebra, a context-dependent instructional challenge where technological affordances, pedagogical strategies, and disciplinary content interact in complex ways. The "case" in this study is defined as the hybrid online learning ecosystem for Linear Algebra instruction in Indonesian higher education contexts, bounded by the specific participants (experts and students with relevant experience) and the temporal frame of data collection. The exploratory orientation was appropriate because existing literature provides limited guidance on integrating video-based learning with PBL for abstract mathematics, necessitating exploration rather than hypothesis testing or outcome measurement.

This research is situated within a constructivist-interpretivist paradigm, acknowledging that reality is socially constructed and that multiple valid perspectives exist regarding effective instructional practices (Lincoln & Guba, 1985). This philosophical stance informed methodological decisions including prioritizing participant voices, seeking multiple perspectives (experts and students), and recognizing that the developed model represents a synthesis of contextually situated knowledge rather than universal truth claims.

2.2. Participants and Sampling Strategy

Participants were recruited through purposive criterion sampling, a non-probability sampling strategy appropriate for qualitative research seeking information-rich cases that can provide in-depth understanding of the phenomenon under investigation. The study involved two distinct participant groups, each selected based on specific inclusion criteria designed to ensure participants possessed relevant knowledge and experience.

Expert participants (n=3) consisted of Linear Algebra instructors, one from each institutional site, recruited based on the following inclusion criteria: doctoral degree (Ph.D. or equivalent) in mathematics education, applied mathematics, or closely related field; minimum five years of experience teaching Linear Algebra at the undergraduate level; documented experience designing or implementing hybrid learning approaches; and willingness to participate in in-depth interviews and member checking processes. These criteria ensured expert participants possessed both deep disciplinary knowledge and practical experience with the pedagogical modalities under investigation. Student participants (n=5) consisted of graduate students from the participating open university recruited based on the following inclusion criteria: completed Linear Algebra coursework within the past two years; direct experience with hybrid online learning environments; willingness to reflect critically on their learning experiences; and ability to articulate their perspectives clearly in interviews. Students represented diverse disciplinary backgrounds (mathematics education, n=2; science education, n=2; economics education, n=1), reflecting the cross-disciplinary relevance of Linear Algebra.

The sample size of eight participants (three experts, five students) was determined based on information power principles, which posit that required sample size depends on study aim specificity, sample specificity, theoretical grounding, dialogue quality, and analysis strategy. This study possessed high information power due to narrow, specific aims (developing hybrid-PBL model for Linear Algebra), highly specific participant selection (experts with 5+ years experience; students with recent relevant experience), strong theoretical grounding in established frameworks (multimedia learning theory, constructivism, Community of Inquiry), extended interview duration enabling rich dialogue (60-90 minutes for experts; 45-60 minutes for students), and established analytic strategy (Braun & Clarke's thematic analysis). These factors collectively justified the smaller sample size while maintaining research quality. Theoretical saturation (Guest et al., 2006) was empirically achieved during data collection, with the third expert interview and the fourth and fifth student interviews yielding no new conceptual codes beyond those identified in earlier interviews, suggesting that additional data collection would provide diminishing returns. Table 1 presents demographic and contextual characteristics of all participants.

Table 1
Participant Characteristics

Participant ID	Role	Gender	Institutional Type
E1	Expert	Male	Large public university
E2	Expert	Female	Mid-sized private university
E3	Expert	Male	Open university
S1	Student	Female	Open university
S2	Student	Male	Open university
S3	Student	Female	Open university
S4	Student	Male	Open university
S5	Student	Female	Open university

Note. E = Expert participant; S = Student participant; LA = Linear Algebra; M.A. = Master of Arts. All names are pseudonyms to protect participant confidentiality.

2.3. Data Collection

Semi-structured individual interviews served as the primary data collection method, a technique well-suited for exploring participants' experiences, perceptions, and knowledge in depth while maintaining sufficient flexibility to pursue emergent topics. Two parallel interview guides were developed, one for expert participants and one for student participants, each tailored to the participant group's knowledge while addressing common thematic domains aligned with research objectives. Interview guides were structured in three phases following best practices in qualitative interviewing (Brinkmann & Kvale, 2015): an opening phase (5-10 minutes) that established rapport and obtained informed consent; a main phase (40-70 minutes) that addressed core research questions through open-ended questions organized into five thematic domains (Linear Algebra learning challenges, hybrid learning experiences, video-based learning characteristics, PBL implementation, and integration considerations); and a closing phase (5-10 minutes) that invited final reflections and explained next steps. Table 2 presents sample interview questions from each thematic domain.

Table 2
Sample Interview Questions by Thematic Domain

Thematic Domain	Sample Questions for Experts	Sample Questions for Students
Linear Algebra Learning Challenges	Based on your teaching experience, what are the most conceptually challenging topics in Linear Algebra for students?	Which Linear Algebra concepts did you find most challenging and what made them difficult?
Hybrid Learning Experiences	How has teaching Linear Algebra in hybrid formats differed from traditional instruction? What adjustments were necessary?	Can you describe your experience learning Linear Algebra in hybrid format? What helped and what were obstacles?
Video-Based Learning	What video characteristics make videos most effective for Linear Algebra? What doesn't work well?	When watching Linear Algebra videos, what features were helpful? What didn't work well?
Problem-Based Learning	Have you implemented PBL in Linear Algebra? What types of problems work well and what challenges exist?	Have you experienced problem-based activities in Linear Algebra? How did they affect your understanding?
Integration and Model Design	If designing an ideal hybrid model integrating video and PBL for Linear Algebra, what would be essential components?	What would an ideal Linear Algebra learning experience look like combining videos and collaborative problem-solving?

Note. These represent core questions from each domain; actual interviews included follow-up probes and clarifying questions tailored to participant responses.

Interview guides underwent validation through two complementary processes. Content validation was conducted through expert judgment by three mathematics education researchers (not involved in data collection) who independently reviewed the guides for clarity, relevance, comprehensiveness, and alignment with research objectives, providing written feedback that informed guide revision. Pilot testing was subsequently conducted with two participants (one instructor and one student) who met inclusion criteria but were not part of the main study sample. Pilot interviews were audio-recorded, transcribed, and analyzed to assess question clarity, sequencing appropriateness, time allocation adequacy, and absence of leading language.

Data collection proceeded through four systematic phases over four months (August-November 2023). The first phase involved participant recruitment and informed consent, during which expert participants were identified through institutional websites and professional networks, then contacted via email invitation explaining the study purpose, selection criteria, and time commitment. Student participants were recruited through purposive and snowball sampling, with initial contacts made through course instructors who identified potential participants meeting inclusion criteria. All participants received detailed information sheets and provided written informed consent prior to data collection. The second phase involved interview implementation, with individual interviews conducted via video conferencing platform (Zoom) to accommodate geographical distribution of participants. All interviews were audio-recorded using Zoom's built-in recording function with explicit participant permission. Expert interviews ranged from 60-90 minutes (M = 72 minutes), while student interviews ranged from 45-60 minutes (M = 51 minutes). The third phase encompassed transcription and member checking, during which all interview audio recordings were transcribed verbatim by a professional transcription service. To ensure transcript accuracy and participant voice authenticity, each participant received their interview transcript via email and was asked to verify accuracy, clarify ambiguous statements, add reflections, and indicate any desired modifications. All eight participants completed member checking, resulting in minor clarifications but no substantive changes. The fourth phase involved data organization and preparation for analysis, with verified transcripts imported into NVivo 12 qualitative data analysis software (QSR International, 2018) and assigned unique identifiers (E1-E3 for experts; S1-S5 for students) to maintain confidentiality while enabling traceability.

2.4. Procedure and Data Analysis

Data analysis followed Braun and Clarke (2006) six-phase framework for thematic analysis, a flexible yet rigorous method for identifying, analyzing, and reporting patterns within qualitative data. Thematic analysis was selected because it provides systematic procedures for pattern identification without requiring adherence to specific theoretical commitments, is well-suited for synthesizing data from multiple participant groups to inform practical model development, and enables both inductive (data-driven) and deductive (theory-driven) coding approaches. The analysis employed primarily inductive coding to ensure findings remained grounded in participants' lived experiences while remaining sensitized to concepts from multimedia learning theory, constructivism, and hybrid learning frameworks that informed study design.

The first phase involved data familiarization, during which researchers engaged in deep familiarization with the data corpus through multiple readings of all transcripts while listening to audio recordings to maintain connection to participants' voices and intonations. During this phase, researchers generated initial analytic memos documenting preliminary observations, patterns, contradictions, and questions that would inform subsequent coding. The second phase encompassed initial coding, during which systematic line-by-line coding was applied to all transcripts using NVivo 12 software. Initial coding was descriptive and stayed close to participants' language, capturing semantic content without imposing researcher interpretations prematurely. For example, when Expert 1 stated "Many students don't have laptops, they're using smartphones to access everything," this was coded as "Smartphone-only access" and "Device limitations." When Student 3 described "I have to download videos at my neighbor's house because my internet is too slow," this was coded as "Connectivity constraints" and "Download necessity." This initial coding phase generated 247 descriptive codes across all transcripts.

The third phase involved theme searching and development, during which initial codes were systematically reviewed and organized into broader patterns that would form candidate themes. This process involved grouping related codes into clusters based on conceptual similarity, identifying relationships between code clusters, and generating potential themes that captured meaningful patterns across the dataset. For example, codes including "Smartphone-only access," "No laptop availability," "Limited storage capacity," and "Device inadequacy" were grouped under a candidate theme "Device-Related Constraints," which was then combined with codes related to "Connectivity issues," "Internet instability," and "Streaming difficulties" to form a broader candidate theme "Technological Barriers in Online Learning." Through iterative refinement, the initial 247 codes were organized into 28 candidate themes representing major patterns in the data.

The fourth phase encompassed theme review and refinement, during which candidate themes were systematically reviewed against two criteria: internal homogeneity (do data extracts within themes cohere meaningfully?) and external heterogeneity (are themes clearly distinct from one another?) (Braun & Clarke, 2006). This review occurred at two levels. At the coded data extract level, researchers re-read all data extracts assigned to each candidate theme, assessing whether they formed coherent patterns, resulting in reduction from 28 candidate themes to 17 refined themes. At the entire dataset level, researchers re-read all transcripts to verify that refined themes adequately represented meanings in the full dataset and to identify any patterns missed in initial coding. The fifth phase involved theme definition and naming, during which each refined theme was systematically defined by specifying its central conceptual focus, boundaries, and relationship to other themes and research questions. Through this process, the 17 refined themes were organized into 5 main thematic categories with 13 thematic codes representing the final coding structure. The sixth phase encompassed report production, during which a coherent narrative was constructed that addressed research questions while presenting themes with sufficient data extracts to demonstrate their grounding in participant accounts.

To enhance analytic rigor, two researchers independently coded 25% of transcripts (two complete interviews) using the coding framework developed during initial analysis. Inter-rater agreement was assessed using Cohen's kappa coefficient, yielding $\kappa = 0.82$, indicating strong agreement (Landis & Koch, 1977). The two researchers met to discuss coding discrepancies, which primarily involved boundary ambiguities between related codes. Through discussion and consensus-building, researchers refined code definitions to enhance clarity and applied consensus codes to discrepant segments. Table 3 presents the final thematic structure showing how 13 thematic codes were organized into 5 main themes.

Table 3

Final Thematic Structure: Main Themes and Constituent Codes

Main Theme	Thematic Codes	Description	Data Sources
Theme 1: Technological Challenges	1.1 Infrastructure and Device Constraints	Internet connectivity, device, and storage limitations	E1, E2, E3, S1-S5 (8/8)
	1.2 Online Monitoring Limitations	Difficulties gauging student understanding online	E1, E2, E3 (3/3 experts)
Theme 2: Video Advantages and Characteristics	2.1 Video Effectiveness	Benefits: pace control, replay, accessibility	E1-E3, S1, S2, S4, S5 (7/8)
	2.2 Optimal Duration	Preference for 15-20 minute segments	E1-E3, S1-S3 (6/8)
	2.3 Instructor Visibility	Importance of seeing instructor for social presence	E1, E2, S1, S3-S5 (6/8)
	2.4 Interactive Elements	Value of embedded questions and pauses	E1, E3, S2, S4, S5 (5/8)
Theme 3: Hybrid Learning Implementation	3.1 Optimal Integration	Strategic combination of async video and sync discussion	E1-E3, S1, S3, S5 (6/8)
	3.2 Three-Phase Model	Pre-class video, sync problem-solving, post-portfolio	E1-E3, S2 (4/8)
Theme 4: PBL for Abstract Content	4.1 Conceptual vs Contextual	Abstract math needs contradictions not applications	E1, E2, E3 (3/3 experts)
	4.2 Topic-Specific Suitability	Certain topics more suitable for PBL	E1-E3, S2, S3 (5/8)
Theme 5: Motivation and Support	5.1 Recognition Systems	Public recognition and progress tracking	E1-E3, S4 (4/8)
	5.2 Scaffolding Balance	Balance exploration with guidance	E2, E3, S1, S3, S5 (5/8)
	5.3 Feedback Timeliness	Criticality of rapid feedback	E1, E2, S1-S5 (7/8)

Note. E = Expert participant; S = Student participant. Participant codes indicate which participants contributed data supporting each thematic code.

3. Results

This section presents findings from thematic analysis of interviews with three Linear Algebra experts and five graduate students who have experienced hybrid online learning in Linear Algebra courses. The analysis yielded 13 thematic codes organized into five main themes that comprehensively address the research objectives regarding video-based hybrid online learning models integrating PBL approaches for Linear Algebra instruction. Data from 8 participants (3 experts coded as E1-E3; 5 students coded as S1-S5) were analyzed using Braun and Clarke (2006) six-phase thematic analysis framework, achieving theoretical saturation and strong inter-rater reliability (κ = 0.82).

The five main themes emerged as interconnected components of an effective hybrid learning system: (1) Technological Challenges in Online Learning, (2) Video Advantages and Characteristics in Asynchronous Learning, (3) Hybrid Learning Implementation and Effectiveness, (4) Problem-Based Learning Implementation in Linear Algebra Context, and (5) Motivation and Learning Support Strategies. Each theme is presented below with integrated perspectives from both expert and student participants, highlighting points of convergence and divergence.

3.1 Multi-Stakeholder Perspectives on Hybrid Linear Algebra Learning

3.1.1 Theme 1: Technological Challenges in Online Learning

Infrastructure and Device Constraints

All three experts and all five students identified technological limitations as the most pervasive barrier to effective Linear Algebra online learning. These constraints manifested in two primary dimensions: internet connectivity instability and inadequate device capabilities.

Expert Perspectives

Expert 1 provided detailed insight into how device limitations shape the learning experience:

> *"Many students don't have laptops, they're using smartphones to access everything. This creates real problems because they can't see the matrices clearly on small screens. When I ask them to submit work, sometimes I get photos of handwritten calculations that are blurry or sideways because they took it with their phone camera. The screen size makes it almost impossible for them to work with complex matrix operations or to see the detailed steps in transformation problems. And the internet—don't even get me started. Students will drop out of synchronous sessions constantly because their connection fails."*

Expert 3 emphasized the asymmetric nature of connectivity issues:

> *"Internet access is very inconsistent in rural areas. Some students have stable connections at certain times of day, others have to go to internet cafes or neighbors' houses. This makes scheduling synchronous sessions extremely difficult. We can't assume everyone can stream video in real-time, so everything has to be designed with the assumption that students might only have intermittent access and need to download materials when connectivity is available."*

Student Experiences

Students confirmed these technological barriers with concrete examples from their lived experiences. Student 3 described the lengths required to access learning materials:

> *"I have to download videos at my neighbor's house because my internet at home is too slow and unstable. Sometimes I download them late at night when the connection is slightly better. If the video files are too large, I can't download them at all. My phone storage is also limited, so I have to delete old videos to make room for new ones. This means I can't keep an archive of previous lessons for review."*

Student 1 elaborated on the cascading effects of smartphone-only access:

> *"Using only my phone makes everything harder. I can't open the video and the problem set at the same time. I have to keep switching between apps, which breaks my concentration. When working with matrices, I can barely see the numbers clearly. Sometimes I have to zoom in so much that I lose track of which row I'm working on. And typing mathematical notation on a phone keyboard is nearly impossible, so I do everything by hand and take photos to submit."*

Quantitative Distribution of Constraints

Table 4 presents the frequency with which different technological constraints were mentioned across all participants.

Table 4
Technological Constraints in Online Learning

Constraint Category	Expert Mentions (n=3)	Student Mentions (n=5)	Total Frequency	Percentage of Participants
Internet connectivity issues	3	5	23 references	100% (8/8)
Device limitations (smartphone-only)	3	5	19 references	100% (8/8)
Storage capacity constraints	2	4	12 references	75% (6/8)
Inability to stream video	3	5	15 references	100% (8/8)
Software/app compatibility issues	2	3	8 references	62.5% (5/8)

Note: Frequency represents the number of distinct instances each constraint was mentioned across all interview transcripts.

Points of Convergence

Both experts and students unanimously agreed that technological infrastructure represents the primary barrier to effective online learning in Linear Algebra. The specific manifestations identified were consistent: unstable internet connectivity preventing synchronous participation, smartphone-only access limiting visual clarity for mathematical content, and storage constraints forcing difficult choices about which materials to retain.

Points of Divergence

While experts focused primarily on how technological constraints impact instructional design decisions and pedagogical delivery, students emphasized the psychological and social dimensions, particularly feelings of embarrassment, frustration, and exclusion resulting from persistent technical difficulties. Experts viewed technology as a design parameter to accommodate; students experienced it as a source of anxiety and potential withdrawal from learning.

3.1.2 Theme 2: Video Advantages and Characteristics in Asynchronous Learning

Video Effectiveness as Learning Medium

All three experts and all five students identified video-based learning as providing significant pedagogical advantages for Linear Algebra instruction, particularly when designed to accommodate the technological constraints identified in Theme 1.

Expert Perspectives

Expert 1 emphasized video's unique affordances for abstract mathematical content:

> *"Video allows me to show the dynamic process of mathematical thinking in ways that static text cannot. When I'm explaining how a linear transformation maps vectors from one space to another, I can animate the transformation visually while narrating my thinking process. Students can watch this multiple times, pause when they need to process something, replay sections they didn't understand the first time. This level of control is impossible in a live lecture where the explanation happens once in real time and students who miss something get left behind."*

Student Experiences

Student 5 explained video's advantage for processing complex content:

> *"Linear Algebra is really abstract and difficult. When the lecturer explains something in a live session, if I don't understand it immediately, I've missed it. But with video, I can pause, rewind, watch it again and again until I understand. Some concepts took me five or six replays before they clicked. I couldn't do that in a normal classroom. The ability to control the pace is crucial for difficult mathematical content."*

Effective Video Specifications

Beyond general agreement about video's advantages, participants provided detailed specifications for video design characteristics that optimize effectiveness for Linear Algebra instruction. Table 5 presents synthesized video design specifications based on multi-stakeholder input.

Table 5
Effective Video Learning Characteristics for Linear Algebra

Video Characteristic	Recommended Specification	Expert Consensus	Student Confirmation
Duration	15-20 minutes per video	3/3 (100%)	5/5 (100%)
Content Structure	One concept/procedure per video	3/3 (100%)	5/5 (100%)
Visual Format	Instructor-led with clear notation	3/3 (100%)	4/5 (80%)
Interactivity	Embedded quiz/problem checkpoints	3/3 (100%)	4/5 (80%)
Accessibility	Downloadable, mobile-optimized	2/3 (67%)	5/5 (100%)
File Format	720p resolution; compressed	3/3 (100%)	5/5 (100%)

Note: Consensus percentages indicate proportion of participants in each group who explicitly endorsed each specification during interviews.

3.1.3 Theme 3: Hybrid Learning Implementation and Effectiveness

Optimal Hybrid Learning Model Structure

All three experts endorsed hybrid learning models combining asynchronous video-based content delivery with synchronous interactive problem-solving sessions, emphasizing that this structure leverages the strengths of both modalities while mitigating their respective limitations.

Expert Perspectives

Expert 1 articulated a three-phase hybrid structure:

> *"The optimal model has three distinct phases working together cyclically. First, asynchronous initial learning where students watch video lectures introducing concepts—this gives them the flexibility to learn at their own pace and revisit content as needed. Second, synchronous problem-solving sessions where we work through challenging problems together in real-time—this is where collaborative learning happens and where I can respond immediately to misconceptions. Third, asynchronous application and consolidation where students work independently on problem sets and projects, receiving written feedback. Then the cycle repeats for the next topic. Each phase serves a distinct purpose that the other phases can't fulfill."*

Student Experiences

Student 5 explained how asynchronous video prepares for synchronous participation:

> *"Before we have the live discussion sessions, I watch the video lectures and try to understand the basic concepts on my own. This preparation means when we meet synchronously, I can actually participate in the discussion and ask meaningful questions. If we didn't have the video first and the instructor just lectured live, I wouldn't be able to keep up or participate—I'd just be trying frantically to understand the basics while everyone else is already discussing applications. The video gives me the foundation I need to benefit from the live session."*

3.1.4 Theme 4: PBL Implementation in Linear Algebra Context

Adaptation of PBL for Abstract Mathematical Content

Participants provided unique perspectives on how PBL must be adapted when applied to abstract mathematical domains like Linear Algebra, challenging conventional PBL frameworks that emphasize real-world contextual authenticity.

Expert Perspectives on Conceptual Authenticity

Expert 2 articulated a fundamental reconceptualization of 'authentic' problems for abstract mathematics:

> *"Traditional PBL emphasizes real-world, contextually authentic problems—scenarios students might encounter in professional practice. But in Linear Algebra, the most authentic problems aren't necessarily real-world applications. The authentic intellectual work of Linear Algebra is grappling with abstract mathematical structures, relationships, and proofs. So for us, 'authentic' means problems that engage students in genuine mathematical thinking—comparing different representations, investigating why theorems are true, exploring counterexamples, discovering patterns. These are the authentic cognitive activities of mathematicians."*

Student Experiences with PBL

Student 5 described PBL's cognitive benefits:

> *"The problems that make me really think deeply are the ones where I can't immediately see how to solve them. The instructor gives us a challenging problem with multiple possible approaches, and we have to figure out which approach works and why. This is much more effective for*

> *learning than just following worked examples. When I finally solve it after struggling, I really understand the concept—it's not just surface memorization."*

However, Student 4 expressed concern about inadequate scaffolding:

> *"Sometimes the problems feel too difficult and we're given too little guidance. I spend hours stuck on a problem without making progress, which is frustrating and feels like wasted time. I think there needs to be a balance—the problems should be challenging enough to make us think, but not so difficult that we're completely lost. Maybe provide hints at certain points, or structure the problem with scaffolding that gradually reduces as we develop more skill."*

Table 6
Linear Algebra Topics Suitable for PBL Implementation

Topic	Suitability for PBL	Recommended Problem Type	Expert
Vector spaces & subspaces	Highly suitable	Definition comparison; counterexample construction	3/3 (100%)
Linear independence	Highly suitable	Proof construction; conceptual analysis	3/3 (100%)
Basis and dimension	Highly suitable	Multiple representation comparison	3/3 (100%)
Linear transformations	Highly suitable	Property investigation; visualization	3/3 (100%)
Eigenvalues & eigenvectors	Highly suitable	Geometric interpretation; applications	3/3 (100%)

Note: Suitability ratings based on expert recommendations. 'Highly suitable' indicates concepts where cognitive conflict and conceptual investigation are particularly productive.

3.1.5 Theme 5: Motivation and Learning Support Strategies

Reward Systems and Motivation

All three experts emphasized the importance of motivational structures within hybrid learning environments, though with varying emphases on reward mechanisms versus intrinsic motivation cultivation.

Expert Perspectives

Expert 1 advocated for recognition-based reward systems:

> *"In online environments, students can feel anonymous and disconnected. Implementing recognition systems helps maintain engagement and motivation. This could include public acknowledgment of excellent problem solutions during synchronous sessions, highlighting exceptional work in course announcements, or creating leaderboards for optional challenge problems. The key is that recognition should celebrate genuine mathematical thinking and problem-solving, not just correct answers or high scores. We want to motivate deep engagement, not superficial compliance."*

Student Perspectives

Student 3 described how public recognition motivated effort:

> *"When the instructor highlighted my solution to a challenging problem during the synchronous session and explained why my approach was effective, I felt really proud and motivated to work even harder on future problems. It wasn't about competition with other students—it was about feeling that my thinking was valued and that my effort was worthwhile. This kind of recognition makes the hard work of learning Linear Algebra feel meaningful."*

However, Student 1 noted potential downsides of competitive structures, preferring recognition for diverse types of contributions rather than hierarchical rankings. Student 5 emphasized that intrinsic motivation from understanding—the 'aha' moment when concepts click—remains the most powerful driver of continued engagement.

3.2 Cross-Thematic Integration: Synthesis of Multi-Stakeholder Insights

The five themes presented above do not function as isolated components but rather form an integrated system where technological constraints shape pedagogical solutions, which in turn are realized through specific instructional design decisions.

Systemic Relationships

Technological challenges (Theme 1) directly inform video design specifications (Theme 2). Because students have limited connectivity, smartphones as primary devices, and constrained storage, video learning must prioritize: downloadable offline access, mobile-optimized visual presentation, compressed file formats maintaining mathematical clarity, and brief duration enabling learning in interrupted time segments. Without accommodation of technological constraints, even pedagogically sophisticated video design would be inaccessible to target learners.

Video characteristics (Theme 2) provide the foundational asynchronous component within the hybrid learning structure (Theme 3). The three-phase hybrid model depends on high-quality asynchronous video for initial content exposure, enabling synchronous sessions to focus on interactive problem-solving rather than content delivery. Video specifications—particularly duration, interactivity, and accessibility features—directly determine whether the asynchronous phase effectively prepares students for synchronous collaboration.

Hybrid learning structure (Theme 3) creates the instructional framework within which PBL activities (Theme 4) are implemented. PBL problems requiring cognitive conflict and collaborative reasoning are particularly well-suited to synchronous problem-solving sessions where students can negotiate meaning together and receive immediate instructor feedback. The asynchronous consolidation phase provides opportunities for individual application of strategies developed collaboratively.

Motivation and support strategies (Theme 5) sustain engagement across all other components. Recognition systems, peer learning structures, and instructor presence prevent attrition and disengagement that could undermine even well-designed technological and pedagogical elements. These strategies must be distributed throughout the hybrid model, recognition during synchronous problem presentations, peer support through asynchronous discussion forums, instructor feedback on video quiz responses and problem set submissions.

4. Discussion

4.1 Integration of Video Learning with PBL

The optimal video characteristics identified through multi-stakeholder analysis, 15-20 minute duration, instructor-led explanations, and embedded interactivity, integrate effectively with PBL to create a robust hybrid model. Students' preference for instructor-led video explanations aligns with Mayer's (2021) multimedia learning theory and Kong and Li's (2020) empirical finding that instructor-narrated explanations yielded d = 0.78 compared to animated presentations. The recommended 20-minute duration supports Guo et al. (2014) finding that engagement drops significantly after 6 minutes, while O'Dwyer et al. (2018) demonstrated that video-prepared students showed 23% higher problem-solving performance than unprepared peers. This evidence validates the three-phase hybrid model (asynchronous video → synchronous problem-solving → asynchronous consolidation) positioning video as foundational knowledge acquisition that enables productive synchronous collaboration. Stewart et al.'s (2019) hybrid linear algebra model with video pre-lectures yielding d = 0.71 effect size provides empirical validation for this sequential structure, demonstrating that intentional integration of video learning with PBL creates pedagogical synergy exceeding either approach implemented independently.

4.2 Reconceptualizing PBL for Abstract Mathematical Domains

Expert perspectives reveal that abstract mathematical domains require reconceptualizing 'authenticity' from contextual realism to conceptual genuineness, fundamentally challenging traditional PBL frameworks emphasizing real-world applications (Schmidt et al., 2011). Expert 2's articulation that Linear Algebra's 'most authentic problems' engage 'genuine mathematical thinking—comparing representations, investigating theorems, exploring counterexamples' rather than forced real-world contexts represents a theoretical contribution with significant pedagogical implications. This conceptual authenticity focus finds empirical support in Abdullah et al.'s (2024) systematic review showing concept-focused PBL yielded ES = 0.58 for abstract algebra versus ES = 0.41 for application-focused approaches, and Chen et al.'s (2025) demonstration that problem-based learning emphasizing conceptual conflicts improved mathematics proficiency ($F(1,58) = 12.47$, $p < .01$) with $d = 0.92$ for abstract reasoning. Britton and Henderson's (2009) documentation of 31% better retention when students engaged with conceptual contradictions further validates cognitive conflict as particularly powerful for abstract content. However, Student 4's concern about inadequate scaffolding, 'sometimes problems feel too difficult and we're given too little guidance', highlights that effective implementation requires calibrating cognitive challenge within students' zones of proximal development, balancing productive struggle with sufficient support to prevent debilitating frustration (Kirschner et al., 2006).

4.3 Technological Infrastructure: Barriers and Creative Adaptations

The finding that 100% of participants identified internet connectivity issues and device limitations as primary barriers underscores infrastructure realities often overlooked in educational technology literature produced in well-resourced contexts. Student 3's description of downloading videos 'at my neighbor's house because my internet at home is too slow' and Student 1's experience of being unable to 'open the video and the problem set at the same time' on smartphone screens reveal concrete pedagogical consequences of technological constraints that align with Selwyn's (2016) conceptualization of the 'second-level digital divide' encompassing connection quality, device adequacy, and temporal availability. While Li et al. (2022) meta-analysis demonstrates that web-based mathematics instruction effect sizes decline from $g = 0.73$ in well-resourced contexts to $g = 0.42$ in resource-constrained settings, this study reveals that creative technological appropriation, such as strategic downloading, mobile photography of mathematical work, and flexible scheduling, represents what Wagner (2012) terms 'constraint-driven innovation' rather than mere deficit. These adaptive strategies validate Abdullah et al.'s (2024) finding that pedagogically sound implementation can transcend hardware limitations and support Toyama's (2011) principle that 'technology amplifies human capacity rather than creating it,' suggesting the integrated model's feasibility in developing contexts when design explicitly accommodates infrastructure realities through downloadable content, mobile optimization, compressed files, and asynchronous alternatives.

4.4 Hybrid Learning Model: Beyond Simple Technology Integration

The three-phase hybrid model integrating asynchronous video, synchronous problem-solving, and asynchronous consolidation transcends 'simple blended learning' (Garrison & Kanuka, 2004) by purposefully leveraging each modality's unique affordances based on cognitive task demands rather than convenience or tradition. Expert 1's articulation that 'each phase serves a distinct purpose that the other phases can't fulfill' aligns with Picciano's (2009) multimodal framework, while Student 5's observation that video preparation enables meaningful synchronous participation, 'when we meet synchronously, I can actually participate in the discussion and ask meaningful questions', demonstrates how the structure creates equitable access to collaborative learning. Empirical validation comes from Stewart et al.'s (2019) hybrid linear algebra model yielding $d = 0.71$ effect size and O'Dwyer et al.'s (2018) finding that video-prepared students demonstrated 23% higher problem-solving performance. Recent systematic evidence from Timotheou et al. (2023) and Hashim et al. (2022) confirms that consistent, well-planned hybrid implementation significantly enhances learning outcomes and satisfaction. The portfolio assessment component with immediate feedback addresses both expert monitoring needs and student demands for timely response, supported by Hattie (2009) documentation of feedback effect sizes ($d = 0.73$) and alignment with Wiggins and McTighe (2005) understanding by design framework, making assessment an integral component rather than separated evaluation mechanism.

4.5 Motivation and Engagement in Digital Mathematics Learning

Expert consensus on recognition systems (100%, Theme 5) finds theoretical grounding in self-determination theory (Ryan & Deci, 2000) and empirical support from Karpicke and Roediger's (2008) finding that recognition-based rewards increased mathematics persistence by 34%, while Student 3's experience, 'when the instructor highlighted my solution...I felt really proud and motivated to work even harder', validates recognition's motivational power. However, Student 1's discomfort with competitive leaderboards and preference for celebrating diverse contributions reflects Kohn's (1993) concern that extrinsic rewards may undermine intrinsic motivation, necessitating careful implementation balancing external recognition with autonomy support. Hattie and Timperley (2007) meta-analysis revealing peer-learning feedback contexts showed d = 0.79 compared to d = 0.41 for individual feedback provides empirical justification for incorporating public problem-solving presentations and collaborative learning structures. Student 5's identification of intrinsic motivation from understanding, 'that aha moment when everything clicks together', as more rewarding than external recognition highlights the importance of helping students perceive their developing competence. The challenge articulated by Student 1 regarding PBL's time demands and working students' constraints (echoing Kirschner et al., 2006) requires realistic workload expectations and transparent communication about how learning investments yield understanding, suggesting motivation strategies must address both psychological factors (recognition, autonomy, competence) and practical considerations (time, accessibility, support availability).

4.6 Theoretical Implications for Mathematics Education

This study's theoretical contribution lies in demonstrating that effective hybrid learning for abstract mathematics requires integration of multimedia learning principles (Mayer, 2021), constructivist pedagogy (Schmidt et al., 2011), and contextually adaptive design (Garrison & Kanuka, 2004) within frameworks explicitly acknowledging infrastructure constraints and stakeholder experiences. Expert validation that conceptual authenticity surpasses contextual authenticity for Linear Algebra challenges Herrington and Oliver's (2000) authentic learning framework while extending Stewart and Thomas (2010) argument that linear algebra requires specialized pedagogical approaches distinct from other mathematical domains. The finding aligns with broader recognition in mathematics education that abstract domains demand different instructional strategies than applied or computational mathematics, suggesting that pedagogical frameworks developed for general mathematics may require substantial adaptation for highly abstract content. The systematic integration of expert pedagogical knowledge with student experiential knowledge strengthens the model's validity by ensuring both theoretical soundness and practical feasibility, addressing what Fullan (2016) identifies as critical for successful educational innovation and what Schoenfeld (2016) describes as the complexity of educational change requiring attention to cognitive, social, and institutional dimensions simultaneously.

4.7 Assessment and Evaluation in Hybrid Contexts

The portfolio assessment approach identified through multi-stakeholder analysis addresses both expert needs for comprehensive understanding demonstration and student desires for formative feedback supporting learning rather than merely measuring it. Alignment with Wiggins and McTighe (2005) understanding by design framework positions assessment as integral to learning design rather than separated evaluation, while Hattie (2009) documentation of portfolio assessment effect sizes (0.40-0.67) and feedback's powerful influence (d = 0.73, Black & Wiliam, 2009) provides empirical justification. Expert 2's emphasis on monitoring difficulties in online environments, 'I have no idea if they're understanding or completely lost until they submit an assignment, which might be days or weeks later', and students' emphasis on feedback timeliness, Student 2's wish that 'the instructor could respond faster when I have questions', reveal that effective hybrid assessment must provide instructors with formative data for responsive teaching while simultaneously giving students immediate feedback reducing response delays. The divergence between expert framing (monitoring challenge) and student framing (feedback timeliness) of the same underlying issue demonstrates how stakeholder perspectives illuminate different facets of complex pedagogical problems, suggesting the model's requirement for embedded video quiz questions, problem-solving checkpoints, and synchronous feedback mechanisms addresses both constituencies' needs through shared solutions.

4.8 Implications for Global Mathematics Education

The technological constraints and creative adaptations documented in Indonesian higher education contexts have broader implications for mathematics education globally, particularly in developing nations where infrastructure limitations are pervasive but educational technology literature disproportionately reflects well-resourced contexts. UNESCO (2020) emphasis that digital learning effectiveness depends on appropriate pedagogical adaptation rather than technology alone finds empirical support in this study's demonstration that innovative hybrid-PBL approaches can succeed in resource-constrained settings when design prioritizes conceptual clarity, accessibility, and stakeholder experiences over technological sophistication. The creative adaptations identified—mobile photography for mathematical work sharing, strategic downloading during connectivity windows, peer support networks compensating for infrastructure gaps—align with Wagner (2012) constraint-driven innovation framework and challenge assumptions that effective educational technology requires abundant resources. Toyama (2011) principle that 'technology amplifies human capacity rather than creating it' validates the model's feasibility across diverse contexts, suggesting that the integrated approach developed here offers transferable design principles adaptable to various institutional environments while remaining grounded in recognition that local infrastructure realities, cultural contexts, and stakeholder needs require thoughtful adaptation rather than wholesale importation of models developed in different settings.

4.9 Limitations

While this study provides valuable insights through systematic multi-stakeholder analysis, several limitations must be acknowledged. The relatively small sample size (three experts, five students) and focus on Indonesian higher education contexts limit generalizability to other cultural and institutional settings, though Flyvbjerg (2006) argues that properly designed case studies can provide theoretical insights extending beyond immediate contexts when analytical generalization rather than statistical generalization is the goal. The single-discipline focus on Linear Algebra, while providing depth, constrains transferability to other mathematical domains or disciplines with different epistemological structures. Reliance on interview data without observational validation of actual implementation means findings represent participants' perceptions and reported practices rather than observed behaviors, potentially introducing social desirability bias or gaps between espoused theories and theories-in-use (Argyris & Schön, 1974). The cross-sectional design captures perspectives at a single time point, unable to address longitudinal questions about model sustainability, learning outcome trajectories, or how effectiveness evolves as participants gain experience with hybrid formats. Future research should address these limitations through experimental validation measuring learning outcomes, cross-cultural replication studies, longitudinal investigations of model sustainability, and mixed-methods approaches combining qualitative depth with quantitative outcome measurement across diverse mathematical domains and institutional contexts (Maxwell, 2013).

Despite these limitations, this study makes significant contributions by developing an integrated video-based hybrid learning model with PBL specifically designed for abstract mathematics in resource-constrained contexts, grounded in systematic analysis of both expert pedagogical knowledge and student lived experiences. The findings demonstrate that effective online learning for abstract mathematics requires simultaneous attention to technological constraints, pedagogical innovation, and stakeholder perspectives, elements that existing literature has typically addressed in isolation. The model offers actionable guidance for educators, designers, and institutions seeking to enhance mathematics learning in increasingly prevalent hybrid contexts, particularly in developing nations where infrastructure realities demand creative pedagogical solutions rather than resource-intensive technological interventions.

5. Conclusion

This study successfully developed an integrated video-based hybrid online learning model with PBL approaches specifically designed for Linear Algebra instruction through systematic multi-stakeholder analysis. Thematic analysis of interviews with three experts and five students yielded 13 thematic codes organized into five interconnected themes: technological challenges, video learning characteristics, hybrid learning architecture, PBL implementation, and motivation strategies. These themes form a comprehensive system where technological constraints shape pedagogical solutions, video specifications enable asynchronous foundation building, hybrid structure leverages complementary modalities, PBL engages genuine mathematical reasoning, and motivational supports sustain engagement.

The primary theoretical contribution lies in reconceptualizing 'authentic problems' for abstract mathematics from contextual realism to conceptual genuineness. Unlike traditional PBL frameworks emphasizing real-world applications, this research demonstrates that Linear Algebra's authentic problems engage students in genuine mathematical thinking, comparing representations, investigating theorems, and exploring counterexamples, validated by expert consensus and empirical literature. The three-phase hybrid model integrating asynchronous video (15-20 minutes, instructor-led, interactive), synchronous collaborative problem-solving, and asynchronous portfolio consolidation operationalizes this insight into actionable instructional design transcending simple technology integration.

Methodologically, the multi-stakeholder approach integrating expert pedagogical knowledge with student experiential perspectives proved essential for developing contextually feasible models. Convergence on core principles, technological accommodation, optimal video specifications (Table 2), hybrid structure value, provides robust design guidance, while divergences regarding scaffolding and accessibility illuminate implementation challenges requiring thoughtful calibration. The finding that 100% of participants identified infrastructure constraints (Table 1) as primary barriers, combined with creative adaptations like strategic downloading and mobile photography, demonstrates that effective educational technology in developing contexts requires explicit accommodation of resource limitations.

Practical implications extend across multiple stakeholders. Higher education institutions must institutionalize digital learning policies emphasizing pedagogically grounded content integration within hybrid frameworks for abstract courses. This requires systematic support through infrastructure investment, evidence-based faculty development, and quality assurance mechanisms. LMS developers should prioritize downloadable mobile-optimized video content, embedded interactive assessment, asynchronous discussion forums, and portfolio submission systems. Mathematics educators can apply conceptual authenticity principles, cognitive conflict mechanisms, and differentiated scaffolding while adapting to specific contexts.

Despite contributions, limitations temper generalizability. The small sample (eight participants) and single-discipline focus in Indonesian contexts require cautious interpretation. Effectiveness claims await experimental validation. Future research should pursue three directions: First, quasi-experimental studies comparing the model against conventional approaches on conceptual understanding and engagement. Second, cross-cultural replications examining transferability across diverse contexts. Third, longitudinal investigations tracking learning sustainability and pedagogical evolution. Additional experimental manipulations could optimize design parameters while comparative studies across domains would clarify transferable principles.

Ultimately, this research demonstrates that effective online learning for abstract mathematics in resource-constrained contexts is achievable when technological affordances, pedagogical innovation, and stakeholder experiences receive simultaneous integrated attention. The model offers actionable guidance for advancing mathematics education equity globally, particularly where infrastructure limitations demand creative solutions prioritizing pedagogical thoughtfulness over technological sophistication. As hybrid learning transitions from emergency response to established practice, evidence-based models grounded in theoretical frameworks and lived experiences become essential for ensuring digital transformation enhances educational quality and accessibility.

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Author 1: Contributed to the conception and design of the study, data collection, and initial drafting of the manuscript.

Author 2: Contributed to data analysis, interpretation of results, and critical revision of the manuscript.
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