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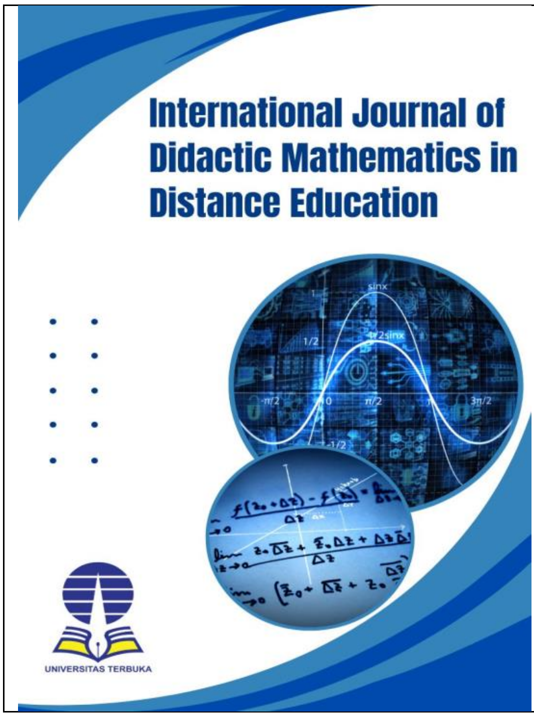

The review of concept image and concept definition: A hermeneutic phenomenological study on the derivative concepts

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The review of concept image and concept definition: A hermeneutic phenomenological study on the derivative concepts

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Abstract
Calculus classes often focus on studying derivatives, a fundamental topic in mathematics. Upon finishing their studies, potential mathematics teachers will educate students about advanced concepts such as derivatives in the classroom. Therefore, comprehending derivative concepts is essential for teaching children effectively. This study aims to determine how potential mathematics teachers view themselves concerning derivative concepts based on their concept image and concept definition. The study utilized a hermeneutic phenomenological approach together with qualitative approaches in its research strategy. The research data was obtained through interviews and clinical tests from six participants from one of the universities in Kuningan Regency, Indonesia. The research findings indicate that participants' concept image of derivative concepts is limited to symbolic representations. Most participants did not view derivative concepts as providing a deeper understanding but rather as a technique to solve procedural problems. The results indicate that utilizing a range of representations in the learning process can improve the development of a more thorough conceptual understanding, leading to better comprehension of derivative concepts.

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1. Introduction

The research in calculus has advanced significantly, exploring people's cognitive processes and educators' viewpoints on effective teaching strategies for knowledge construction (Park, 2015; Thompson, 1994). The derivative is a challenging concept in calculus due to its definition and the need to grasp related concepts such as functions, difference of quotients, and limits (Arnal-Palacián & Claros-Mellado, 2022; Park, 2015; Rivera-Figueroa & Ponce-Campuzano, 2013; Thompson & Silverman, 2008; Thompson, 1994). The derivative is a crucial term in numerous scientific areas that deal with changes and fluctuations in quantities (Fuentealba et al., 2018; Park, 2015; Dubinsky et al., 2000). Derivative comprehension is a perennial problem for colleges and universities when it comes to teaching mathematics at the university level (Bressoud, 2015). Previous research findings suggest that the knowledge taught and learned regarding derivative ideas focuses on grasping the meaning of these concepts (Aspinwall et al., 1996; Bezuidenhout, 1998; Orton, 1983). This discovery aligns with a study by Aspinwall et al. (1996) that identified the phenomena where individuals can easily understand a function but struggle to apply and explain the findings in different situations. Hence, to get all-encompassing information for resolving various problems, Zandieh (Dubinsky et al., 2000) presents a framework for testing students' understanding of derivative notions via several representations.

According to the National Council of Teachers of Mathematics (NCTM), the word representation includes both actions and outcomes that are vital to helping people understand mathematical ideas (Ferrini-Mundy, 2000). Visual representations of the derivative may include the slope of the tangent line at a given location or, when enlarged, the slope of a line that seems to approach the curve. Speed, the limit of the difference quotient, and the instantaneous rate of change are other physical and verbal descriptions (Borji et al., 2018; Tokgöz, 2012). According to Zandieh (Dubinsky et al., 2000). Table 1 provides a framework for investigating derivative ideas in various forms.

Table 1

The Framework of Derivative Concepts

Process-object layer	Representations				
	Graphical	Verbal	Paradigmatic Physical	Symbolic	Other
	Slope	Rate	Velocity	Difference Quotient	
Ratio					
Limit					
Function					

The multiple representations are provided by this outline, which can be used as a measuring tool to assess conditions such as an individual's understanding of mathematical concepts that are accepted by the mathematics community, the effectiveness of teaching strategies that introduce different aspects of a concept, the effectiveness of curriculum-aligned teaching methods, and the essential concepts covered in a carefully planned curriculum (Borji et al., 2018; Moru, 2020; Tokgöz, 2012). The slope of the line that a magnified curve seems to approach or the tangent line's slope at a given point are two ways to picture the derivative. It can also be described as the instantaneous rate of change, or in physical terms, as speed or velocity. Symbolically, it is represented as the limit for the difference quotient. Representation typically refers to the presence of a fundamental structure that is depicted in every setting (Dubinsky et al., 2000).

This framework allows other scholars to investigate derivative concepts' definitions due to the variances that can arise in different contexts (Park, 2015; P. W. Thompson, 1994; Vinner, 1983; Vinner & Dreyfus, 1989). Within the realm of functions and geometry, the concept of the derivative is defined by Giraldo et al., (2003) as the slope of the tangent line to the graph of the function $y = f(x)$ at a point $(a, f(a))$. The derivative is the change in the value of f(x) as a function of x. The derivative is a concept in physical science that deals with the rate of change of an object's velocity in response to a constant force. The function $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{x}$ is the formal definition of the derivative of $f(x)$. Along with comparable representations such as $\frac{\Delta y}{\Delta x}$ or $\frac{dy}{dx}$, the idea of limits is included within the framework of the instantaneous rate of change. Multiple studies looking at the meaning of derivative ideas support Zandieh's statement (Dubinsky et al., 2000). According to the studies conducted by Borji et al.

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(2018), different representations are shown by individuals as they acquire derivative concepts.

Cognitive structure-building research (Bressoud, 2015; Duval, 2006) uses concept definition and idea picture analysis as a theoretical foundation for interpreting findings. Understanding mathematical concepts depends less on commonly accepted formal definitions and more on representations of ideas, according to Vinner & Dreyfus (1989). The person's understanding of a mathematical idea may be evaluated by using mental constructs such as a concept image and a concept definition (Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). The difference between a mathematically defined concept and a person's mental representation of that idea is the focus of concept image and concept definition. The mental representation, attributes, and operations of ideas, including words, symbols, and pictures connected to mathematical concepts, make up what is known as the concept image. The idea picture includes concept definitions, which give a more in-depth explanation or interpretation of a concept via the use of different terms (Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989).

Individuals in Indonesia often solve ordinary problems without being able to offer conceptual meaning based on connections between concepts like gradient, function, limit, and continuity, according to multiple studies (Desfitri, 2016; Destiniar et al., 2021; Mufidah et al., 2019; Prihandhika et al., 2018). According to Nurwahyu et al. (2020) and Prihandhika et al., (2020), this condition might hinder learning since it makes it hard for people to solve issues in contexts that are different from what they were taught. Individuals frequently resolve mundane problems without understanding the conceptual significance based on connections between concepts such as gradient, function, limit, and continuity, according to studies conducted in Indonesia at both the secondary and tertiary levels (Desfitri, 2016; Destiniar et al., 2021; Mufidah et al., 2019; Prihandhika et al., 2018). Because of this condition, people may struggle to learn new things since they are unable to apply their prior knowledge to new situations (Nurwahyu et al., 2020; Prihandhika et al., 2020).

Problems in understanding and presenting concepts could be at the root of people's difficulties in learning (Giraldo et al., 2003; Tall & Vinner, 1981). A person may build an internal representation of a notion, according to Edwards & Ward (2004), by rote memorization of a formal definition, even if they don't grasp the idea. As a result, the individual's conceptual picture may clash with the formal definition of the notion. To better connect conceptual descriptions with formal concept definitions when teaching derivative concepts, and to decrease the likelihood of problems with solving conceptual problems, it is necessary to devote extra time and effort to mastering important concepts, essential concepts that will be taught, and teaching strategies (Park, 2015; Sbaragli et al., 2011). Based on the definitions and concept images provided in the background and literature studies, this study aims to assess how participants describe derivative ideas.

2. Method

The study's methodology is qualitative and based on a hermeneutic phenomenological approach; it delves deeply into the participants' lived experiences to shed light on and analyze the relevance of phenomena, attitudes, beliefs, perceptions, and ideas (Creswell, 2015). Previous research has shown that students' knowledge of derivatives at the school and university levels is mostly procedural. This study uses concept image theory to determine whether participants' grasp of derivative ideas matches with that. dealing with fundamental problems without understanding specific representations embedded in derivative concepts (Desfitri, 2016; Destiniar et al., 2021; Mufidah et al., 2019; Prihandhika et al., 2018). Participating in the research were six undergraduates from a university in West Java, Indonesia, who were preparing to become mathematics teachers. Purposive strategies were used to choose participants based on predetermined criteria (Creswell, 2015). Everyone taking the class has to have finished differential calculus during the first semester. Using the framework proposed by Zandieh, a test with four questions about the idea of derivation was administered to gather data for the study (Dubinsky et al., 2000). The following topics were covered in the questions: 1) how well participants could derive functions; 2) how well they understood that the gradient of a tangent line is the first derivative of a function; 3) how well they understood that the derivative represents instantaneous speed in physics; 4) how well they understood that the derivative is an approximation value of the difference quotient equation of a function. To confirm the test results, clinical interviews were conducted to determine how the participants understood the generated ideas. These interviews included both easy and complicated questions (Hunting, 1997). Data reduction, data presentation, and conclusion drafting are three parallel processes that process research data acquired from interviews and testing (Creswell, 2015).

Concept image theory is utilized to analyze study findings regarding the representation of derivative concepts in participants' cognitive structures during the process of drawing inferences (Tall & Vinner, 1981; Vinner & Dreyfus, 1989). The elements of concept image consist of: 1) mental image: information imagined by participants in a cognitive structure based on acquired experience for use in explaining concepts and designing procedures to solve problems. 2) procedure: the syntax chosen by participants to translate mental images for problem-solving. 3) properties: axioms, definitions, lemmas, theorems, formulas, or mathematical rules used in explaining concepts and solving problems (Tall & Vinner, 1981). The concept image components are subjective and will evolve based on the information and experience acquired by each individual in teaching. Thus, idea representations can depict varying results when comparing different individuals. Participants' conceptual pictures can be seen and examined through mental images, methods, and features elucidated in responses to questions posed by researchers during tests and clinical interviews (Nurwahyu et al., 2020). The research design involved examining participants' cognitive processes in detail to identify different meanings and representations based on the concept image and concept definition that each participant held for derivative concepts. Furthermore, the study design aids in interpreting the diverse occurrences uncovered during the research. The research findings cannot be generalized to a larger and more diverse population due to potential variations in the concept image of prospective mathematics teacher students across different locations.

3. Results and Discussion

The first inquiry in the data collection process was to investigate the participant's idea image when asked to derive a function. All participants

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demonstrated deriving the function $f(x) = x^2 + x - 4$ using the formula $f'(x) = n.x^{n-1}$ and obtained the answer $2x + 1$ during the first problem-solving session. During the confirmation stage of the function derivation process, all participants envisioned two key properties in their mental images: the power rule $f'(x) = n.x^{n-1}$ and the limit rule $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$. The limit rule was not selected as a method for finding a solution due to its more complex solution steps in comparison to the power rule in calculus. The participants' idea image of the form $f'(x) = n.x^{n-1}$ was shown to be quite dominant based on the approach employed to create a function.

The second question exam attempts to assess participants' specific understanding of several topics connected to derivatives, particularly focusing on geometric representations such as functions, equations of lines, gradients, and tangent lines. Four participants defined procedures to solve problems, while two participants successfully applied procedures to find solutions using relevant properties of formal concepts. They utilized formulas such as $m = \frac{\Delta y}{\Delta x}$ or $m = \frac{y_2 - y_1}{x_2 - x_1}$, for determining gradients, $y - y_1 = m\ (x - x_1)$ for finding the equation of a line from one coordinate, and $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$ for the equation of a line based on two coordinates. They also calculated the equation of the tangent line to the curve $f(x) = x^2$ using the formula $y_1 = m\ (x - x_1)$ where $m = f'(x)$. The third question's examination seeks to ascertain the participant's conception of concepts particularly speed that are drawn from paradigmatic physical representations. In the third question, three things are wanted to be observed: the participants' comprehension and interpretation of the instantaneous speed at t seconds, the maximum stationary point or still point in the function function $s(t) = -t^2 + 12t$, and the average speed over a certain time frame. In response to questions on average speed, five participants calculated speed by comparing changes in time and distance using a problem-solving process with the parameters $v = \frac{s}{t}$ or $v = \frac{\Delta s}{\Delta t}$. Additionally, three participants utilized the approach to respond to questions on instantaneous speed that involved the nature of a derivative concept—that is, speed at $t = 4$. Two participants utilized the attribute $s'(t) = 0$ to find the maximum height or stationary point of an item in the previous question about the stationary point feature of functions. In the meanwhile, four players solved the issue by entering the value of t seconds into the function right away, disregarding the necessity for a stationary point. Finding out how the participant understands derivatives using the function's approximation of $\frac{\Delta y}{\Delta x}$ is the end objective of the test.

These calculations may help establish a mental understanding of derivatives by based on the relationship between the notions of limit and comparison. But no matter how many observations were made, not a single person could determine the optimal course of action based on the expected traits to handle the scenario. Based on these findings, it seems that the participants' mental image of how linguistic representations of derivative concepts' rates of change are still evolving.

The clinical interviews were the next step in the study process; these interviews allowed us to probe the participants' understanding of the derivative concept's representations, learn more about their mental process, and verify their concept image using the participants' definitions of the concept. One method for keeping tabs on, assessing, and making sense of unusual mathematical behavior is clinical interviews (Hunting, 1997). For the purpose of eliciting responses and revealing the subject's underlying cognitive processes, clinical interviews often include open-ended questions and activities (Heng & Sudarshan, 2013). In order to investigate the mental pictures formed by participants' idea representations, clinical interviews were administered to all of them. The interview process presentation only features two participants, who illustrate the patterns and uniqueness of replies about generated conceptions. The reason for this is that researchers found it tough to probe deeper into the outcomes of the clinical interviews since each participant provided a confined concept image. The figure depicts an interview technique with the researcher (R), the first participant (P1), and the second participant (P2) involved. What follows is a presentation of the results from the clinical interviews with P1 and P2.

R : "What exactly is a derivative, and could you please explain it to me?"

P1 : "The idea of the derivative is to lower the power value of a function; for instance, if my function $f(x) = 2x^3$, its derivative is $6x^2$. One advantage of the derivative notion is that it allows us to calculate the value based on speed."

R : "What is the process for getting $6x^2$?"

PS1 : "The way to do this is by using this formula, sir. $f'(x) = nx^{n-1}$. If the function is $f(x) = 2x^3$, then the derivative will be $f'(x) = 3.2x^{3-1}$. So, the number three is raised to the power, down to the next to be multiplied by 2, and later the number three to the power is also reduced by 1. The result is $6x^2$"

R : "Did you use $f'(x) = nx^{n-1}$. to determine the derivative of $f(x) = x^2 + x - 4$?"

P1 : "Yes, that's right sir, I used this formula to solve the derivative of the function $f(x) = x^2 + x - 4$. From the results of the reduction, we get the result $f(x) = 2x + 1$"

R : "Is there an alternative formula that can be used to obtain the reduction results for this function?

P1 : yes, there is, sir, using the formula $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, but I don't know how to derive a function using the formula. It seems like a difficult and long process, sir."

R and P1 in a clinical interview

R : "How is the definition of the concept of derivative based on your understanding?"

P2 : "This concept is symbolized by $f'(x)$ or $\frac{dy}{dx}$ to derive the value of a function, sir"

R: "Are there any other symbols that you know?"

P2 : "Nothing, sir. I only know those two symbols."

R: "Do you know the meaning of the symbol that was conveyed earlier?"

P2 : "The symbol $f'(x)$ means the function is reduced once, sir. Related to $\frac{dy}{dx}$, it is the change in the y value relative to the x value.

R : "Is there anything else that you understand from the definition of the concept of derivative?"

P2 : "as far as I know, the concept of derivative is defined using the limit and slope of the tangent line, Pa. So the first derivative of a function is a line that touches a curve, as long as the limit exists. It is denoted by $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

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R and P2 in a clinical interview

The results from clinical interviews with P1 and P2 showed that the concept-object function layer, in particular, made heavy use of the concept image derivative from the concept definition. Derivatives are defined in S1 and S2 as a way to derive functions from their original values, using the formula $f'(x) = n.x^{n-1}$. The formula $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ is also included in the mental representations of P1 and P2. Nevertheless, P2 stresses that it is unable to derive a function using this attribute as a process. P1 presented a concept definition of the derivative concept using the paradigmatic physical representation of the process-object layer ratio in relation to speed after P2 and P1's mental pictures were confirmed by asking follow-up questions on the concept's meaning. According to P1, physics speed concerns may be addressed by using the notion of derivatives. In the meanwhile, P2 illustrates the idea's description in terms of tangent gradients by means of a graphical depiction of the border between the process and object layers. The formula $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ is derivative from the idea of tangent gradient and the expression $f'(x) = n.x^{n-1}$ is derivative from the preceding formula, as shown in P2. Using the concept definition from Table 2, the following is a depiction of the derivative concepts in P1 and P2.

The concept image based on the concept definition given was found to be very dominating in symbolic representation, particularly in the process-object function layer, according to the findings of clinical interviews with S1 and S2. According to S1 and S2, the idea of a derivative is used to derive a function using the formula $f'(x) = n.x^{n-1}$. In their mental pictures, S1 and S2 likewise possess the property of the formula $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. S2 makes clear that it is unable to derive a function using this attribute as a method. Following up with queries on the meaning of the derivative concept, S1 confirmed the mental pictures of S1 and S2, and then S1 used the paradigmatic physical representation of the process-object layer ratio in the context of speed to express a concept definition of the derivative idea.

S1 clarified that speed issues in physics may be resolved by applying the derivatives approach. In the meanwhile, S2 uses a graphical depiction of the process-object layer boundary in the context of tangent gradients to explain the concept definition. The formula $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ is derivative from the idea of tangent gradient, as explained in S2, and the expression $f'(x) = n.x^{n-1}$ is derivative from the development of the preceding formula. Based on the concept definition presented in Table 2, the derivative concepts provided in S1 and S2 are represented as follows

Table 2

Representation of Derivative Concepts Understood by S1 and S2

Process-object layer	Representations				
	Graphical	Verbal	Paradigmatic Physical	Symbolic	Other
	Slope	Rate	Velocity	Difference Quotient	
Ratio					
Limit	○		○		
Function				●	

The results of research shows that people grasp derivative ideas best when they see them represented symbolically within the context of speed and the process-object layer. In order to figure out how to handle issues procedurally, the participants' mental pictures include the information of $f'(x) = n.x^{n-1}$. Other difficulties, particularly conceptual ones, may become more difficult to solve as a result of this circumstance. Answers to questions concerning gradients in graphics and speed in paradigmatic physical representations showed that only a small percentage of participants could identify the solution procedure that was pertinent to the formal concept definition. Beyond that, clinical interview data demonstrated that participants' grasp of the derivative as a tool to reduce a function's number of powers remained restricted. Our results show that the participants' mental models of the derivative ideas are lacking in the representations necessary for their comprehension.

As a means to enhance individual knowledge thoroughly, the use of multiple representations to teach derivative ideas is strongly stressed (Asiala et al., 1997; Aspinwall et al., 1996; Bezuidenhout, 1998; Orton, 1983). The idea being taught may not have been communicated as effectively if the dominance of one representation in understanding derivative notions causes problems. The assertion is backed by research by Rivera-Figueroa & Ponce-Campuzano (2013), who found that students in differential calculus classes frequently fail to understand how to use the derivative—the slope of a tangent line—to find the solution to the equation $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$. Furthermore, Borji et al. (2018) found that in their country, symbolic representations are usually given more weight than graphical representations while teaching calculus. The results seem to be consistent with those of previous studies done in different countries (Moru, 2020; Park, 2015). Most students do well on routine issues when education focuses on a single representation, but they have a hard time with problems that need knowledge of several contexts, which is known as conceptual knowledge. This is especially the case when trying to decipher the relevance of derivatives in different situations and their connection to the difference quotient limit (Baker et al., 2000; Moru, 2020).

4. Conclusions

The qualitative research conducted using a hermeneutic phenomenological approach aims to investigate participants' understanding of learning derivative concepts, specifically focusing on concept images, concept definitions, and their meanings. A study on potential mathematics teacher candidates at a university in West Java yielded findings consistent with prior research. Challenges in teaching and comprehending derivative concepts, as previously discussed, continue to be a prevalent occurrence in various locations. Conducting a follow-up evaluation of how concept images are formed and how participants interpret concept definitions from different representations is a crucial step in generating alternate answers to common research challenges. Tall & Vinner (1981) identified various potential individual responses during the concept image formation process, such as 1) substituting the existing concept image with new information or knowledge; 2) retaining the old concept image, or utilizing both independently; the concept image aids in comprehending the formal concept definition. The research findings indicate that the complexity of an individual's concept image significantly enhances the depth and breadth of knowledge of the issue under study. Researchers and practitioners should focus on teaching strategies at school and university levels to engage a diverse group of individuals in transforming conceptual knowledge into formation. In future studies, it is advised that researchers develop learning designs using concepts obtained from concept images and concept definitions to enhance students' learning experiences.

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Declarations
Author Contribution :
Author 1: Conceptualization, Writing - Original Draft, Editing and Visualization, Validation
Author 2: Writing - Review & Editing, Formal Analysis, and Methodology

Conflict of Interest:
The authors declare no conflict of interest.

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