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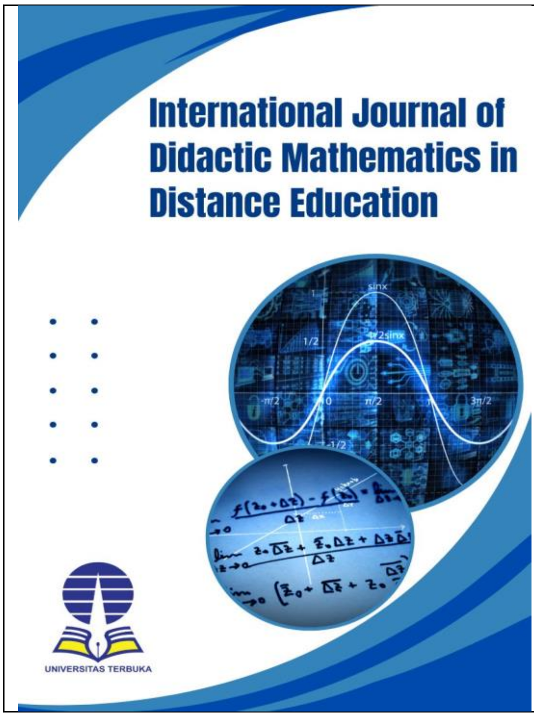

Pupils' didactical Mileu in an inclusive discourse in basic triangle geometry

Clement Ayarebilla Ali[1], **Emmanuel Asemani**[2], **Michael Tangkur**[3]

[1]Department of Basic Education, University of Education, Winneba, Ghana, ayarebilla@yahoo.com

[2]Department of Basic Education, University of Education, Winneba, Ghana, easemani@uew.edu.gh

[3]Department of Educational Foundations, Simon Diedong Dombo University of Business and Integrated Development Studies, Ghana

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Pupils' didactical Mileu in an inclusive discourse in basic triangle geometry

Clement Ayarebilla Ali[1*], Emmanuel Asemani[2], Michael Tangkur[3]
[1]Department of Basic Education, University of Education, Winneba, Ghana, ayarebilla@yahoo.com
[2]Department of Basic Education, University of Education, Winneba, Ghana, easemani@uew.edu.gh
[3]Department of Educational Foundations, Simon Diedong Dombo University of Business and Integrated Development Studies, Ghana, mtangkur@gmail.com

[*]Correspondence: aliclement83@gmail.com

Abstract
The concept of inclusivity is the practice of placing pupils with disabilities in normal classrooms. However, the idea of the triangle continues to pose fundamental challenges. This study sought to explore didactical mediation on the learning of the triangle beyond the classroom. The research design used mixed methods sequential exploratory to help identify the pupil's triangle interpretation. The researchers first conducted a semi-structured interview on their experiences in learning the triangle both in school and at home. The responses were transcribed and coded in themes. The themes that emerged were properties, types of angles, and applications to everyday life. The researchers then went further to test the pupils' knowledge on the same themes. The findings showed that didactical mediation with the cultural artifacts, signs, and language largely explained the emergence of the four main properties and must be targeted for the theory, policy, and practice of inclusive education. The prototype testing also confirmed that even though pupils with hearing impairment obtained higher scores, both groups obtained higher mean scores on the post-test than did the pre-test. This indicated that the learning achievement was significantly higher in the post-test than in the pretest due to the didactical mediation. It was recommended that the triangle's learning and teaching should not be limited to academic exercises but to real-life situations. Also, not visually friendly mathematics concepts should be explored to uncover the most suitable methods and techniques to make inclusive mathematics learning a reality for all groups of disabilities.

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1. Introduction

Traditionally, inclusivity, also called mainstreaming, is the practice of placing pupils with disabilities into normal classrooms, either for all (i.e., 'full inclusion') or for most of the time (in which case, pupils are 'pulled out' from the classroom from time to time for occupational or physical therapy, speech/language pathology, or other related services) (Buhagiar & Tanti, 2011). The general idea is to promote inclusive discourses in which several pupils with disabilities learn side-by-side with age peers who have no disabilities.

This means inclusion is no longer a matter of 'allowing' disabled pupils to be in mainstream schools as long as they can fit into the existing systems and cultures (Ali, 2021). Instead of simply integrating them into a mainstream setting, inclusion is about bringing change in school policies, practices, and attitudes so that disabled pupils achieve integration on their terms (Ahmed, 2020). One can therefore argue that rather than preparing pupils to be ready to enter the mainstream school and be able to stay there, in inclusion it is the school that has to ready itself for increasingly diverse students. Ali (2021) identified problems of blind students' learning of equations. Leton et al. (2019; 2020) even included persons with hearing impairments. This study adds geometry to examine the sensibility of the two impairments towards mathematical shapes.

Inclusion hinges on addressing social injustices and parallel discourses such as human rights and equal opportunities. Inclusion should touch on equity, participation, community, entitlement, compassion, respect for diversity, and sustainability (Aljundi & Altakhayneh, 2020). The journey or struggle for inclusion has been like the confluence of different streams of thought – social, political, and educational – that have been gaining momentum to bring inclusion ever closer (Buhagiar & Tanti, 2011).

Inclusion is both a pedagogical issue and a rights issue. Despite the progress made, there is still much to be achieved to enable inclusion at the classroom level, exactly where it matters most (Buhagiar & Tanti, 2011; Madungwe, 2020). Ghana has currently implemented policies about inclusion (Ministry of Education, 2019) and still needs to fill some gaps. Instead of giving up on inclusion and its derivatives at school and classroom levels, it is frequently regarded as not possible to move to total inclusion in all schools. Truly, the journey to inclusion requires time, willingness, and commitment, it must be delayed without justifiable reasons (Buhagiar & Tanti, 2011). However, this paper sets the pace and new norma for comparing pupils with various disabilities

Some Mathematics Persons with Impairments

Research (AMS, 2002) shows that many countries have produced and continue to produce mathematicians with impairments. One of the greatest mathematicians ever, Leonhard Euler (1707–1783), produced around 850 works. Another great person is Nicholas Saunderson (1682–1739), who became the Lucasian Professor of Mathematics at Cambridge University. We cannot forget these two Russians, Lev Semenovich Pontryagin (1908–1988), who invented the mathematical symbols for set intersection, and A. G. Vitushkin's works in complex analysis. France produced Louis Antoine (1888–1971), who proved the three-dimensional analog of the Jordan-Schönflies theorem, and J. W. Alexander (1924), who came up with wild embedding of the two-sphere in three-space.

The list is inexhaustive. However, there is low participation in mathematics by persons with visual impairments. Of course, civilizations have never been the same but there is always more room for people to learn from other cultures. It is critical to start having discourses in mathematics for pupils with hearing and visual impairments in developing countries (Ministry of Education, 2019). It is believed that didactical situations are one of the best ways to help pupils develop interest and curiosity to learn mathematics and take up their careers.

Besides visual, hearing-impaired pupils also have simple arithmetic skills in performing oral calculations using sign language. The deaf children used the same general types of strategies that are used by hearing children (i.e., modelling, counting, and fact-based strategies) (Leton et al., 2019). They do not use as much of their cognitive structures as the normal ones. They use more of their visual prowess which gives them better opportunities than visually impaired ones (Ali, 2021).

Didactical Situation in Specific Conditions

Education of persons with visual impairments aims to integrate these persons among the sighted ones in the common schools. These are specific conditions that must find different new relations in the classroom. This means the whole teaching process changes in comparison to the classic whole-class situation. One has to conceptualize how to communicate between the teacher, the content, and the visually impaired pupils. This triad was especially suitable for the Theory of Didactical Situations.

The Theory of Didactical Situations was introduced by Brousseau in 1997 (Brousseau, 2002). To Brousseau, when acquiring new knowledge in the frame of a didactical situation, teaching a new notion consists of setting up its situations and carrying out interactions in which the learner can participate. This interaction is largely specific to the knowledge being taught in the form of adidactical situations (Brousseau, 2002; Kohanová, 2006).

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Consequently, the researchers present the didactical triangle in Figure 1 to illustrate the system of relationships (didactical contract) between the mediating artifact, the subject (pupils), and the object (triangles) that coexist in the didactical milieu.

Figure 1

Activity Theory (adopted from Kohanová, 2006)

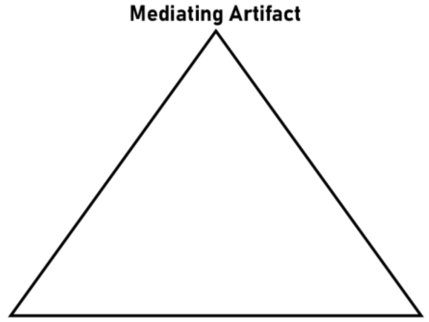

Figure 1 presents the activity theory framework deemed suitable for this study. The activity theory is one of the cultural-historical schools of psychology founded by Vygotsky, Leontiev, and Luria, which is related to the tetrahedron. This tetrahedron completely transcends the situations of the concept of artifact-mediated and object-oriented actions. In the framework, the unit of analysis is composed of a subject, and an object, and mediated by a tool. The subject is the visually impaired pupil, and the object is being held by the subject and motivates the activity in a specific direction. The mediation occurs through the use of many different types of material tools as well as mental tools, including culture, ways of thinking, signs, and language (Kohanová, 2006).

The Concept of Geometry and the Triangle

Geometry is one area of mathematics for teaches about lines, line segments, rays, angles, and geometric shapes. The contents are developed to enable visually impaired pupils to learn how to identify points, lines, angles, types, properties, and applications of triangles (Pritchard & Lamb, 2012). Almost all other geometry concepts require a drawing that cannot be used on a Braille typewriter. Sudirman et al. (2023; 2023) analyze epistemology in a 3D geometric thinking framework reviewed from the aspects of representation, spatial structure, and measurement. Kuswara et al. (2024) analyzed online didactic design for fraction reduction through lesson study activities.

Empirical studies show that Zahra et al. (2018) interviewed persons with impairments about their interpretation of two-dimensional shapes according to their thinking. They explained that the visual experience of a pupil with total blindness from birth has an important role in the way they illustrate or describe two-dimensional shapes. They normally explore the shape and the length by using tactual ability through their sense of touch. Jurmang (2015) also found that the newly adapted teaching strategy developed gave access to learners with visual impairment to participate in geometry and it enhanced the task performance of the learners in geometry. Again, Sibiya & Mudaly (2018) investigated the effects of the Geoboard on learners' understanding of geometric theorems and discovered that Geoboards improved learners' understanding of geometric theorems, especially understanding of geometric terminology and reasoning. However, there is a gap in their comparison of blindness and deafness in mathematics.

Manipulatives remain effective tools for teaching geometry to visually impaired pupils by helping to capture their explanation of the geometric concept for discovery or illustration. There are several teaching tools available on the market that can serve this purpose effectively. However, the imported products are too expensive. Therefore, teachers need to implement certain cultural techniques to provide persons with impairments with an effective

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mathematics education. Mathematics teachers in special schools need geometry teaching tools for impaired persons (Pritchard & Lamb, 2012; Junthong et al., 2020).

Andriyani et al. (2018) discovered that persons with impairments take a long time to construct a mental representation of spatial concepts. The difficulty of learning geometry is not only experienced by the pupils but also experienced by teachers who teach this subject. Although using tools in physical models, many teachers have difficulty instructing geometry to persons with impairments who cannot use visual sense.

On the other hand, the difficulty of pupils with impairments in geometry is also caused by the limitations of geometry books in Braille format as well as geometry learning media. The difficulty of gaining access to information is still experienced by persons with visual impairments as a minority. Even though they are granted access to information as their normal counterparts, several experiences of difficulty do not mean eliminating opportunities for them to study geometry (Ali, 2021).

Governments all over the world are making frantic efforts to promulgate mathematics curricula to suit both impairments and normal pupils. However, visually impaired pupils need specific strategies to learn and teach geometry and triangles (Andriyani et al., 2018). The question of how persons with disabilities can classify triangles is interesting for the researcher. From the interpretation of this study, it is expected that persons with disabilities discover their obstacles so that teachers can find strategies and learning media to help salvage them. Again, it is expected that persons with impairments can identify how the abstract concepts of the triangle and its properties have been related to real objects that they encounter every day (Andriyani et al., 2018).

Even though we live in a modern era, the learning process for persons with disabilities is still based on old-fashioned techniques. These techniques retard their progress and performance in triangles in particular. People with good sight still have huge advantages over their disabled counterparts because the sighted can access books, computers, and other gadgets. On the other hand, persons with impairments cannot have access to these tools. Therefore, other procedures must be developed (Sorathia, 2021). Persons with visual impairments learn Braille for reading, use audiobooks for gathering information, and most importantly, develop the tactile sense which they can feel and learn shapes, objects, and much more. However, their fate still hangs in the balance when it comes to geometric concepts especially those of the triangle (Sorathia, 2021).

To benefit from the learning and teaching of geometry and triangles, an exploration into the importance and applications of the triangle to visually impaired people is a forgone conclusion. The results depend also on the effort the visually impaired persons make to learn how to apply triangles. A good starting point is learning the basic concepts of the triangle, namely the types of triangles, properties of the triangles, and applications of the triangle (Zahra et al., 2018). In the same way, pupils learn how to draw lines so that they can write letters and numbers afterward, persons with disabilities have to learn how to draw lines and basic shapes to further develop other important abilities and capabilities. This way, visually impaired people will analyze the triangle and identify the essence of the concept in many facets of human life (Sorathia, 2021).

Research Questions

The researchers formulated the following research questions on triangles' properties, types, and applications for the visually-impaired pupils:

1) What properties uniquely differentiate the triangle from other plane figures?
2) How do pupils classify the triangle?
3) How can pupils apply knowledge of the triangle to everyday life situations?

2. Method

2.1 Design

The research was a mixed methods sequential exploratory design where qualitative data collection, analysis, and discussion preceded quantitative (Creswell & Creswell, 2018). In the dominant qualitative phase, a narrative qualitative approach was adopted to identify the persons with disabilities interpretations of triangular shapes. The set of interviews consisted of questions about the subject's interpretation of triangular shapes, especially right, scalene, isosceles, and equilateral. Participants of ten and nine pupils with visual and hearing impairments were purposely chosen as a source of qualitative data.

Data on the visually impaired pupils' interpretations of the triangular shapes were collected using task-assisted interviews and the interviews were carried out by using two equivalent sets of tasks at two different times. Every impaired pupil was assigned to describe the types of triangles that they know and give their properties and applications to real life. When a pupil mentioned the triangles, they were further probed to illustrate the example in their own way, by using three similar sticks of the same color, texture, and length (Andriyani et al., 2018). After the interviews, test questions on the triangles were administered to confirm and corroborate the interview responses.

2.2 Participants

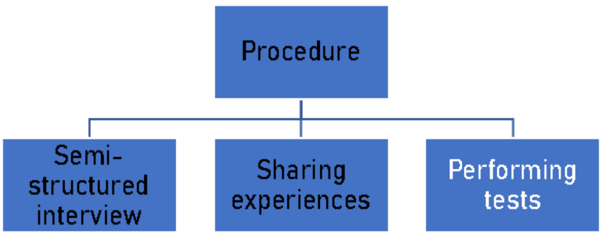
The impaired pupils entered the University of Education, Winneba in the Department in the 2019/2020 academic year. There were 10 and 9 pupils with visual and hearing impairments. These were made up of 6 females and 13 males. The researchers communicated with the pupils and sought permission to conduct this study with the pupils abiding by the ethics of research. All participants had learned rays, lies, and line segments by drawing or at least by illustrating these concepts with manipulatives as part of the Ghanaian curriculum for tertiary education policies (Ministry of Education, 2019).

2.3 Data Collection Procedure

The researchers conducted a semi-structured interview with the participants (See Figure 2). The participants were asked about their experiences in learning triangles both in the classroom and at home and their interest in not only continuing to learn triangles but to applying the concepts in their daily lives. They then shared their experiences in learning in the classroom and their opinion regarding the need for teaching triangles and other geometric concepts to students with visual impairment and blindness (Andriyani et al., 2018).

Figure 2

The Data Collection Procedure



The participants were encouraged to answer the questions very briefly and directly with examples if necessary as it was their first experience with learning mathematics at the University. Later, the researcher transcribed the statements collected from the participants by dividing their verbal feedback into smaller themes for easier and simpler analyses. Thereafter, the researchers administered test items to confirm their experiences (Sorathia, 2021).

Research Junthong et al. (2020) showed that the requirements of mathematics classes at many schools for persons with impairments were obtained using semi-structured interviews. The information revealed that the most difficult content was geometry and that the teaching tools were not sufficiently effective for visually impaired students. The braille books were not good enough for the pupil's understanding because students needed real materials/objects to improve their visual imagery. This is because visually impaired students equally need 3D objects to touch and feel to enhance their learning.

Research by Junthong et al. (2020) also found that the old wooden geoboards in the mathematics classes were large, heavy, colorless, and hard to use. There were carved grid lines on a plane with square edges and circular edges. Due to the difficulty of wooden geoboard making and carrying, a new proposal has replaced the existing model with an unlimited design of light and colorful manipulatives. It is the backdrop of these challenges that the researcher created several patterns and colors for prototype testing. Plastic sticks were designed to form solid objects to ease demonstrations and discussion. The sticks had grid arrays and triangular shapes glued end to end as if they were molten at the vertices to mimic the various triangles. The molten sticks of triangles were distributed to every visually impaired pupil (Junthong et al., 2020). The colorful creations of the materials enhanced the learning activities for visually-impaired pupils. They were so light and portable to handle and twist in any direction when demonstrating for types, properties, and applications. To be successfully applied and used with the visually impaired pupils, they were made to mimic the braille scales with columns of three dots. In most cases, the instruments had holes at the vertices stacked with ribbons (Junthong et al., 2020).

Quite aside from these creations, a set of plastic mathematical sets contained teaching accessories, such as rulers, protractors, set squares, and arrowheads. Two of the set squares were 30-30-60- and 45-45-90-degree triangles. These helped the visually impaired pupils to grasp the concepts at alarmingly faster rates, as well as apply the concepts to other geometric concepts (Junthong et al., 2020).

2.5 Data Analysis

The data collected from the activities and interviews were then classified, reduced, and validated by using triangulation methods to yield credible data. The credible data were analyzed by using qualitative research analysis including verbal transcriptions, narrations, interpretations, and comments. Thereafter, quantitative research analysis followed to measure their level of understanding of the triangle (Creswell & Creswell, 2018) using a One-sample t-test and a Paired Samples Test.

3. Results and Discussion

3.1. Results

The analysis in this part has been divided into the four themes. The first theme was the properties, the second was the classifications by angles, the third was classifications by sides and the fourth was applications of the triangle, contains the analysis of the qualitative data, and the second section contains the analysis of the quantitative data.

The Concept and Properties of the Triangle

Researcher: How do you understand the concept of a triangle?

Excerpt 1: *A triangle is a closed, two-dimensional shape with three straight sides.*

Excerpt 2: *A triangle is a polygon and the basic polygon.*

Excerpt 3: *A triangle has three sides, three angles, and three vertices.*

Excerpt 4: *The sum of all internal angles of a triangle is always equal to* ***180°*** *This is called the angle sum property of a triangle. The sum of all exterior angles of any triangle is equal to* ***360°***

Excerpt 5: *The sum of the length of any two sides of a triangle is greater than the length of the third side. Similarly, the difference between the lengths of any two sides of a triangle is always less than the length of the third side.*

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Excerpt 6: *The side opposite to the largest angle of a triangle is the largest side. Similarly, the side opposite to the smallest interior angle is the shortest side.*

Excerpt 7: *Any exterior angle of the triangle is equal to the sum of its interior opposite angles. This is called the exterior angle property of a triangle. The height of a triangle is equal to the length of the perpendicular dropped from a vertex to its opposite side, and this side is considered the base.*

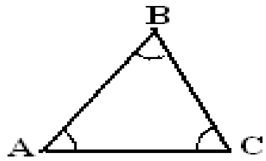
Clearly, the responses show that B the pupils with visual impairments grasped the concept of the triangle so well. Beginning from the number of sides and angles, the pupils extended the learning sums of interior and exterior angles. This initial knowledge of the pupils builds a formidable foundation for higher complex concepts of the triangle.

Classifications of the triangle based on side, angle, and set squares

Researcher: How do you classify triangles based on their sides?

Figure 3

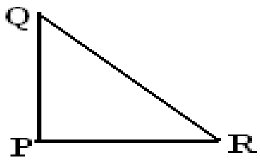
The acute triangle



Excerpt 8: *Acute Triangle: A triangle whose all the angles are acute (< 90^0) is called an Acute-Angled Triangle or Acute Triangle* (see Figure 3).

Figure 4

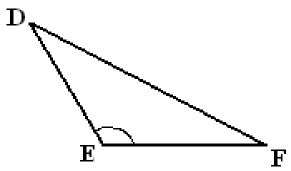
The right triangle



Excerpt 9: *Right Triangle: A triangle whose one angle is a right angle (= 90^0), is called a right-angled triangle or right triangle. A triangle cannot have more than 1 right angle*. In ΔPQR or ∠QPR = 90^0(see Figure 4).

Figure 5

The obtuse triangle



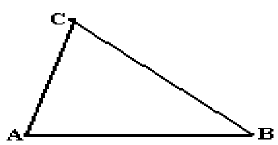
Excerpt 10: *Obtuse Triangle: A triangle whose one angle is obtuse (≥90^0) is called an Obtuse-Angled triangle or Obtuse Triangle. That is, a triangle can not have more than one obtuse angle. In ΔDEF, ∠DEF is an obtuse angle and the other two angles are acute angles* (see Figure 5).

In this question, the pupils adequately addressed the issues. They even predicated one essential property on the size of a triangle 'no angle can be up to 180°'. This revelation was essential since the sum of interior angles is already 180°.

Researcher: How do you classify triangles based on their angles?

Figure 6

The scalene triangle



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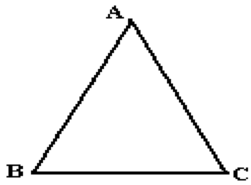
Excerpt 11: *Scalene Triangle: A triangle whose all sides are of different lengths is called Scalene Triangle*. In the figure, we can see that AB ≠ BC ≠ AC (see Figure 6).

Figure 7

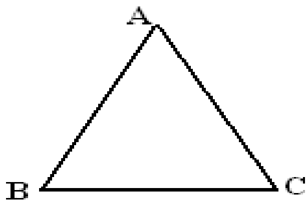
The isosceles triangle



Excerpt 12: *Isosceles Triangle: A triangle whose any two sides are equal is called an Isosceles Triangle*. Here, in the ΔABC, two sides, AB = AC are equal (see Figure 7).

Figure 8

The equilateral triangle



Excerpt 13: *Equilateral Triangle: A triangle whose all sides are equal is called an Equilateral Triangle*. In the ΔABC, all the three sides are equal, AB = BC = AC (see Figure 8)

The pupils correctly classified the three main types of triangles based on angles. Having been able to use artifacts to portray the sizes of the angles was even more intriguing. It is possible that the pupils could further relate the lengths of the artifacts to the angles of the triangles.

Researcher: What is the relationship between the angle and the side of a triangle"

> Excerpt 14: *Equilateral triangle: An equilateral triangle is a triangle whose all three sides are equal.* Here, XYZ is an equilateral triangle as XY = YZ = ZX.
>
> Excerpt 15: *Isosceles triangle: An isosceles triangle is a triangle whose two sides are equal*. The adjoining figure shows an isosceles triangle where XY = XZ.
>
> Excerpt 16: *Scalene triangle: In a scalene triangle, all three sides are unequal*. The figure shows a scalene triangle where XY ≠ YZ ≠ ZX.

Excerpts 14 to 16 show that pupils could adequately relate the lengths of the sides to the types of triangles. However, the names of the angles were conspicuously missing. This vacuum needed to be probed further.

Researcher: Which type of triangle gives you an interesting discovery?

> Excerpt 17: *45-45-90 triangle: Two angles measure 45°, and the third angle is a right angle*. The sides of this triangle will be in the ratio 1: 1: √2 respectively. This is also called an *isosceles right-angled triangle* since two or angles are equal.
>
> Excerpt 18: *30-60-90 triangle: This is a right-angled triangle, since one angle = 90°. The angles of this triangle are in the ratio 1: 2: 3, and the sides opposite to these angles are in the ratio 1: √3: 2 respectively. This is a scalene right-angled triangle since all three angles are different.*

In an attempt to clarify Excerpts 14 to 16, the pupils landed on the special angles. This was a clear indication they had a moderate understanding of the relation between the lengths of the sides and the angles of the triangle. However, the excerpts were limited to the isosceles and scalene right-angled triangles. It was therefore necessary to quiz pupils on the daily encounters with the triangles. This would ensure that they extend their knowledge beyond the isosceles and scalene triangles.

Applications of triangles to daily life situations

Researcher: Outline one practical significance of learning the triangle.

Excerpt 19: *Traffic signs: Traffic signs are in equilateral triangular shape. This means that all three sides are of equal lengths and have equal angles*.

Excerpt 20: *The Roof: The roofs of the houses are made in a triangle shape. The roof truss is an obtuse-angled triangle. In this type of triangle, any one of the three angles is more than 90 degrees. The roof truss is constructed because it does not let water or snow stand on the roof for a longer time*.

Excerpt 21*: Staircase and ladder: The construction of the staircase involves knowledge about right angles*. The staircase is built in a triangular shape, mostly in a right-angle triangle. Moreover, the ladder when placed against the wall at any angle also makes a triangle.

Excerpt 22: *Towers: Many buildings are constructed in triangular shapes to make them more appealing and beautiful. We can notice that towers are triangular. The triangular shape gives strength to the tower since it forms a strong base*.

Excerpt 23: *Pizza Slices: Most of us start our day with triangular sandwiches*.

We can observe that mothers make sandwiches in triangular shapes to make them look more appetizing and because of the triangular shapes, the sandwiches come in handy. Studies show that triangularly shaped sandwiches are more preferred by pupils than the ones that are non-triangular in shape.

The study of descriptive statistics related to visually-impaired and hearing-impaired students in geometry education provides valuable insights into their learning needs and challenges. By analyzing data on their performance, educators can better understand how these impairments affect spatial reasoning and geometry comprehension. This analysis is crucial for developing inclusive teaching strategies that accommodate diverse learners and enhance their engagement with geometric concepts (See Table 1).

Table 1

Descriptive Statistics of visually-impaired, and hearing-impaired Geometry

Impairment	Minimum Statistic	Maximum Statistic	Mean Statistic	Std. Deviation Statistic
Visual	6.00	31.00	15.5000	8.10007
Hearing	6.00	22.00	15.8889	6.03002

In Table 1, the minimum value for persons with both visual and hearing impairments was 6.00. The maximum values were 31.00 and 22.00 respectively. However, the means and standards of 15.50(8.1) and 15.89(6.0) showed that statistics for persons with hearing impairments were better than for persons with visual impairments.

Next, after analyzing that the data is normally distributed, a one-sample t-test was conducted to compare the results of individuals with visual and hearing impairments. This test aims to determine whether there are significant differences in performance between the two groups, providing insights into the impact of their impairments on geometry learning outcomes. These results required that we test the one-sample t-test to compare the results of persons with visual and hearing impairments (See Table 2).

Table 2

One-sample t-test

Group	Test Value = 0					
	t	Df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
Vision	6.051	9	.000	15.50000	9.7056	21.2944
Hearing	7.905	8	.000	15.88889	11.2538	20.5240

In Table 2, a one-sample t-test was run to determine whether persons with visual impairments' score in the triangle was different from normal, defined as a score of 0. The base value was the significance level of .005. The scores were normally distributed, as assessed by Shapiro-Wilk's test ($p > .05$) and there were no outliers in the data, as assessed by inspection of a boxplot. The mean score (15.50 ± 0.00) was much higher than the normal score of 0, a statistically significant difference (95% CI, 9.71 to 21.29), $t(9) = 6.051$, $p = .000$). In other words, persons with hearing visual impairments performed well in the learning of the triangle.

Concurrently, another one-sample t-test was run to determine whether persons with hearing impairments' score in the triangle was different from normal, defined as a score of 0. The scores were also normally distributed, as assessed by Shapiro-Wilk's test ($p > .05$) and there were no outliers in the data, as assessed by inspection of a boxplot. The mean score (15.89 ± 0.00) was even much higher than the persons with visual impairments, a statistically significant difference (95% CI, 11.25 to 20.52), $t(8) = 7.905$, $p = .000$). So, persons with hearing impairments performed better than persons with visual impairments in the learning of the triangle.

Table 3
Paired Samples Test

Impairment	Paired Differences					t	df	Sig.
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
Visual -Hearing	-2.11111	9.54521	3.18174	-9.44821	5.22599	-.664	8	.526

In Table 3, a paired samples t-test showed that the participant's level of learning the triangle increased from visual impairments ($M = 15.50$, $SD = 8.10$) to hearing impairments ($M = 15.89$, $SD = 6.03$; $t(8) = -.664$, $p > .001$, $d = -.22$). Since the p-value is greater than the significance level $\alpha = 0.05$, we fail to reject the null hypothesis. Thus, we had sufficient evidence to suggest that the mean scores were different for both the persons with visual and hearing impairments in the learning of the triangle. This difference could have only been explained by the sensitivity of the human senses towards geometry and nothing more.

3.2 Discussion

The differences in the mean scores between the pre-test and post-test of the two groups were analyzed using independent two-sample t-tests. The significant scores were compared between the experimental and the control groups to evaluate their learning achievement. The quality of the tools and questionnaire were checked by experts. The knowledge of the visually impaired pupils was then determined using test scores (Junthong et al., 2020). There were three main issues for consideration: properties of the triangle, types of triangles, and applications of triangles. Indeed, the questionnaire was similar to that of the interview guide but there were differences in a few items. The data on the pretest-posttest and knowledge on the triangle were collected after the implementation of the semi-interview guide (Junthong et al., 2020).

The analysis of the properties of the triangle showed four main standpoints. The first point is the sum of interior and exterior angles. While the sum of the interior angles is always 180◦, the sum of the exterior angles is always 360◦. Another important property is the sum of two interior angles is the opposite exterior angle. This makes it possible to find the third interior angle. Equally important is that the side opposite to the largest angle of a triangle is the largest side and the side opposite to the smallest interior angle is the shortest side. This is important to help the visually impaired trace the size of an angle once the measure of the side is known (Andriyani et al., 2018).

The fourth property is the height of a triangle is equal to the length of the perpendicular dropped from a vertex to its opposite side, and this side is considered the base. This property is particularly important for the subsequent determination of the area and perpendicular height of the triangle (Kohanová, 2006). The commonest classifications of triangles are acute angles, obtuse angles, right angles, and reflex angles. However, the side classification still exists and these include equilateral, isosceles, scalene, and right triangles. As the classifications reach a crescendo, we can obtain the 30-30-60 and 45-45-90 degree special triangles from only the right-angled triangle (Ali, 2023). These classifications form the basis of trigonometry (Junthong et al., 2020).

The applications of the triangles range from transportation, and construction to food and nutrition. The roofs of the houses are made in an obtuse-angled triangle in order not to hold rainwater or snow. This prevents rushing and decoloration of roofing sheets. Traffic signs are in an equilateral triangular shape to help motorists visualize in all directions, the staircase is right angle triangle to help in smooth inclination and declination, towers are constructed in triangular shapes to make them more appealing and beautiful, and more especially pizza slices are triangular to make them look more appetizing and handier (Andriyani et al., 2018).

4. Conclusion

The theory of didactical situations in the tetrahedron began with the artifact-mediated and object-oriented actions of the triangles. Pupils as subjects, and triangle concepts as objects were successfully and effectively mediated by tools. The mediation occurred in the cultural artifacts, signs, and language. This largely explained the successive transitions. In the mediations, it was inferred that the persons with hearing impairments could explain the properties of the triangle better than the persons with visual impairments. The four main properties dominated the discussion and must be targeted for theory, policy, and practice. The prototype testing showed that hearing-impaired pupils obtained higher mean scores on the post-test than did the pretest, indicating that the learning achievement was significantly higher in the post-test than that of the pretest.

The participants' satisfaction in learning the properties of the triangle was evaluated more on the post-test than the pretest scores. These properties were considered more effective for helping them understand the triangle with good visual imagery than when using traditional tools. The pupils enjoyed learning to trace and mentally create didactic relationships in the triangle. The relatively low results notwithstanding, it was most intriguing to observe how the visually impaired categorized the applications of the triangle. From constrictions to transportation, and from roofing to feeding, the shapes of triangles prominently featured. This is a clear indication that learning and teaching of the triangle is not limited to academic exercises but to real-life situations. We therefore recommended that mathematics concepts that are not visually friendly be explored. This will enable stakeholders to explore the most suitable methods and techniques to make inclusive mathematics learning a reality for all.

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Author Contribution

Author 1: Conceptualization, Writing - Original Draft, Editing and Visualization;
Author 2: Writing - Review & Editing, Formal analysis, and Methodology;
Author 3: Collecting data, Validation and Supervision.

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Conflict of Interest
The authors declare no conflict of interest.

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