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Investigating ontogenic and didactical obstacles vocational high school students face in solving counting rules problems

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Abstract

Research on the ontogenic and didactical obstacles vocational high school students face in solving counting rule problems has been limited. Ontogenic obstacles involve students' readiness, while didactical obstacles concern the sequence and presentation of instructional materials. Therefore, teachers must consider both when developing effective materials. This research aims to describe the ontogenic and didactical obstacles experienced by vocational high school students in solving counting rule problems. The participants of this study were 24 grade XII students from a public vocational high school in Cirebon. Using a Didactical Design Research (DDR) approach, this qualitative study focuses on the prospective analysis stage, examining these obstacles. The research instrument included five diagnostic essay questions and interview guidelines. The results revealed that students encountered instrumental-type ontogenic obstacles, particularly in understanding factorial operations. Additionally, didactical obstacles arose from incomplete teaching materials and improper notation, which led to learning barriers. This research offers valuable insights for teachers and researchers in designing instructional materials, especially for counting rules in vocational schools.

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1. Introduction

Vocational High School is a secondary school that prepares students with technical skills to enter specific jobs (Murtiyasa et al., 2020). The counting rules are one of the subjects taught in vocational high school. The counting rules is a topic that needs special attention in mathematics subjects, especially for vocational high school students. This is because counting rules have numerous problem-solving contexts, so solving counting rules problems requires a deep mathematical reasoning ability, critical thinking skills, logical reasoning, and intelligent insight (Maher et al, 2011; Lockwood, 2013; Lockwood, 2015; Lockwood, 2016). Other fields that use the counting rules include computer science, communication, genetics, and statistics (Eizenberg & Zaslavsky, 2004; Lockwood, 2015). The benefits of studying the counting rules include teaching students how to count, make predictions, generalize, and think systematically (Kapur, 1970). Counting rules are used to calculate the number of possible outcomes in various situations, which is essential for understanding probability. For example, knowing the probability of defective products in quality control can help determine the necessary inspection procedures (Walpole et.al, 1993).

The topic of counting rules at the vocational high school level includes material on the multiplication principle, addition principle, factorial notation, permutations, and combinations. Understanding the rules of counting in the counting rules material is a challenge for students, especially in determining when and how to apply combinations, permutations, and the multiplication rule to calculate the number of possibilities of an event (Wroughton & Nolan, 2012). To solve counting rules problems, accurate interpretation is required in understanding the given problems, so not all types of questions can be directly applied to formulas.

Several research results show that students still experience difficulties in solving counting rules problems. Some studies show that students at various levels (from high school to university) less than 50% of students can correctly answer basic counting rules problems (Lockwood, 2011; Melusova & Vidermanova, 2015; Lockwood & Gibson, 2016; Eizenberg & Zaslavsky, 2004; Godino et al., 2005). Vocational high school students are known to be unable to solve problems, use problem contexts in calculations, and are unable to provide explanations for mathematical calculations in solving counting rules problems (Adhimah & Ekawati, 2020). Vocational high school students experience difficulties understanding concepts and problems and making calculation procedure errors when solving counting problems (Nugroho, 2021).

Two steps to teach this material easily are understanding students' difficulties in solving combinatorial problems and identifying variables that might be causing these difficulties (Batanero, 1997; Lockwood, 2013). Understanding and identifying students' difficulties in solving counting rules problems are done by analyzing learning obstacles. A learning obstacle is not a lack of knowledge but rather a piece of knowledge acquired by individuals when learning mathematics that originates from external sources (Brousseau, 2002). Brousseau (2002) defines three types of learning obstacles: ontogenic, didactical, and epistemological.

Ontogenic obstacles arise due to the difference between the levels of knowledge of the students and the teachers. These are divided into several types, including psychological (students' unpreparedness to learn due to psychological aspects such as low motivation and interest in the material being studied), instrumental (technical problems that prevent students from fully following the learning situation due to a lack of understanding of technical aspects), and conceptual (difficulties related to the conceptual level contained in a design that is incompatible with the students' state based on their previous learning experiences) (Suryadi, 2019a). Ontogenic obstacles can be identified by interviewing students to assess their awareness of the benefits of learning certain materials in daily life (Nopriana et al., 2023; Isnawan & Alsulami, 2024; Kuswara et al., 2024).

Didactical obstacles are caused by the lack of appropriate methods or approaches used by the teacher in teaching. Didactical obstacles can be identified by conducting documentation studies and interviews with teachers and students about the teaching materials used during the learning process (Nopriana et al., 2023). Lastly, epistemological obstacles occur due to limited knowledge of a particular context, which makes it difficult or impossible to apply the knowledge when faced with a different context (Septyawan & Suryadi, 2019; Prihandika & Perbowo, 2014).

Several studies have been conducted to identify students' learning obstacles in solving problems related to sequences and series, statistics, and probability (Rachma & Rosjanuardi, 2021; Dewi et al., 2022; Augie et al., 2023; Sari et al., 2024; Magfiroh et al., 2024). However, there is still little research that discusses the analysis of learning obstacles faced by vocational high school students, particularly in solving counting rules problems. Yet, understanding students' learning obstacles is crucial before designing teaching materials. Analyzing students' learning obstacles benefits teachers and other researchers in designing teaching materials that can minimize students' learning obstacles (Rachma & Rosjanuardi, 2021; Dewi et al., 2022). Additionally, research focusing on analyzing ontogenic and didactical obstacles is still limited, although these types of obstacles are frequently encountered by students. Ontogenic obstacles are difficulties that arise due to the limitations of students at their developmental stage. On the other hand, didactical obstacles emerge from the sequencing and stages of the curriculum, including its presentation in the classroom (Suryadi, 2019b). They involve mismatches between the actual learning process (sequence and stages of presentation) and the theoretically intended stages. Therefore, an initial analysis of the ontogenic and didactical obstacles faced by vocational high school students is crucial for teachers when developing instructional materials for counting rules. Based on the explanation of the importance of counting rules for vocational high school students and the obstacles faced by students in solving counting rule problems, it is necessary to analyze the learning obstacles faced by students, especially ontogenic and didactic obstacles, to achieve learning success. So that this research question is divided into two descriptions of how ontogenic obstacles and didactic obstacles are experienced by vocational high school students in solving counting problems.

2. Method

This study uses a qualitative research method with a Didactic Design Research (DDR) approach. DDR is research that reveals learning obstacles in education and aims to anticipate and eliminate these obstacles (Suryadi, 2019a). In this study, the researcher aimed to uncover the ontogenic and didactical obstacles faced by students. Therefore, the DDR research paradigm used in this type of study is known as the interpretive paradigm. DDR research consists of three stages: (1) prospective analysis, (2) metapedadidactic analysis, and (3) retrospective analysis, which links the results of the didactic situation analysis with the results of the metapedadidactic analysis (Suryadi, 2013). This study was conducted at the prospective analysis stage. In this study, the researcher involved 24 students from the 12th grade majoring in Computer and Network Engineering at a public vocational school in Cirebon City. The researcher served as the primary instrument in the study. In collecting data, the researcher used several additional instruments, including diagnostic tests for learning obstacles in the counting rules material, which consisted of four essay questions. Additionally, the researcher prepared interview guidelines to confirm students' responses in their answer sheets and to explore the ontogenic and didactical obstacles experienced by the students.

The stages of the research conducted based on the prospective analysis of DDR include: (1) selecting and reviewing literature on the material to be studied, in this case, the counting rules, (2) preparing research instruments, including diagnostic tests for students' learning obstacles and interview guidelines for teachers and students, (3) identifying learning obstacles related to the teaching of the counting rules. The tests were given to students who had prior learning experiences with the counting rules at the vocational high school level, (4) conducting interviews to confirm the types of learning obstacles students faced in solving counting rules problems, (5) conducting a documentation study based on teaching materials and students' learning notes on the counting rules material, (6) analyzing ontogenic and didactical obstacles based on diagnostic tests, interviews with teachers and students, and the documentation study of students' learning notes.

3. Results and Discussion

3.1 Results

3.1.1 Ontogenic Obstacle

Based on their nature, ontogenic obstacles are divided into three types: psychological, instrumental, and conceptual ontogenic obstacles (Suryadi, 2019b). In this study, the researcher focuses on the instrumental ontogenic obstacles experienced by students. Instrumental ontogenic obstacles are a type of barrier faced by students in learning or mastering a particular skill or activity. These obstacles relate to the lack of access to tools or instruments necessary for performing a task or activity. In the context of learning, instrumental ontogenic obstacles can occur when students do not have access to or the ability to use the tools or instruments needed to learn a particular skill or topic. For example, if students want to learn programming skills but do not have access to a computer or the required software, this can be an instrumental ontogenic obstacle for them.

The research findings show that students experience instrumental ontogenic obstacles in performing basic multiplication operations and factorial operations. Permutation and combination problems are closely related to multiplication operations, and students still face difficulties in performing multiplication calculations, as well as decomposing and operating factorial values. This leads to incorrect problem-solving. A detailed description of the instrumental ontogenic obstacles experienced by vocational high school students is provided in the discussion section on ontogenic obstacles.

The instrumental obstacles faced by vocational high school students in solving combinatorial problems are illustrated in Figure 1.

Figure 1
 Ontogenic Obstacle Type 1

b. Uraikan banyaknya cara menentukan banyaknya susunan kepengurusan OSIS yang mungkin dibentuk!

ketar

I	II	III
M	T	B

banyaknya kandidat = 9
 yang dipilih = 3

ketar banyaknya
 ke-1 = 9-1 = 8
 ke-2 = 8-1 = 7
 ke-3 = 7-1 = 6

Jadi : $8 \times 7 \times 6 = 122$ cara

Students experience instrumental type ontogenic obstacles, especially in calculating the multiple probability of several events.

The first type of instrumental ontogenic obstacle was found when students performed multiplication, specifically the calculation of $8 \times 7 \times 6$. As shown in the figure, the student first correctly multiplied $8 \times 7 = 56$. However, when the student attempted to multiply 56×6 using vertical multiplication, instead of multiplying, the student added the numbers. This mistake in multiplying tens and units should no longer occur at the high school level.

Figure 2
 Ontogenic Obstacle Type 2

b. Buatlah model matematika untuk menghitung banyaknya cara menentukan susunan kepengurusan OSIS yang mungkin dibentuk!

$$P(9,3) = 9! (9-3) = (9 \cdot 8 \cdot 7 \cdot 6) \cdot 6 = 9 \cdot 8 \cdot 7$$

$$P_r^n = \frac{n!}{(n-r)!} \text{ atau } n! (n-r)! \quad (1)$$

c. Apakah Selesaikan perhitungan dari model matematika yang telah dibuat pada bagian b!

$$P(9,3) = 9! (9-3) = (9 \cdot 8 \cdot 7 \cdot 6) \cdot 6 = 9 \cdot 8 \cdot 7 = 504$$

The second type of instrumental ontogenic obstacle (Figure 2) was identified in part b, where the student used the formula $P_r^n = n! (n - r)!$. It is known that the correct permutation formula is $P_r^n = \frac{n!}{(n-r)!}$, not $P_r^n = n! (n - r)!$. He student expanded $P(9,3) = 9! (9 - 3)! = 9! \times 6!$. The students did not write the factorial sign and made mistakes in expanding the multiplication of the factorial value. This represents an ontogenic obstacle of the instrumental type, related to the students' ability to operate with factorial numbers.

Figure 3
 Ontogenic Obstacle Type 3

$$nC_r = \frac{n!}{(n-r)! r!}$$

$${}^{13}C_2 = \frac{13!}{(13-2)! 2!} = \frac{13!}{11! 2!} = \frac{10! 3!}{10! 1! 2!}$$

$$= \frac{1 \times 2 \times 3}{1! 1 \times 2}$$

$${}^{13}C_2 = \frac{3}{1}$$

The student believed that the factorial notation $13!$ could be expanded into $10!$ multiplied by $3!$. Similarly, they expanded $11!$ into $10!$ multiplied by $1!$.

The third type of instrumental ontogenic obstacle (Figure 3) occurs when students expand factorial notation. As shown in the figure, the student believed that the value of $13!$ could be expanded into $10! \times 3!$.

The same mistake occurred when expanding the value of $11!$ into $10! \times 1!$. In the context of solving the problem, the student aimed to simplify the factorial operation. However, the student should have expanded $\frac{13!}{11!2!} = \frac{13 \cdot 12 \cdot 11!}{11! \cdot 2} = \frac{13 \times 12}{2} = 13 \times 6 = 78$. Based on this explanation, the student should have expanded $13!$ into $13 \times 12 \times 11!$ to simplify it with the divisor, which is $11!$. The value $13!$ is not equal to $10! \times 3!$. The student's error in operating factorials is a type of instrumental ontogenic obstacle that can be fatal in permutation and combination calculations. The student may understand the concept of permutations and combinations well enough to determine the formula or rule to use. However, if this third type of instrumental ontogenic obstacle appears when solving problems involving factorial operations, the student will incorrectly determine the final result of the problem.

The instrumental ontogenic obstacles experienced by vocational high school students in solving counting rules problems are partly due to a lack of understanding of basic mathematics. Before learning permutations and combinations, students must first understand basic mathematical concepts such as factorials, simple combinations, and permutations. If students do not have a good grasp of these fundamental concepts, it will be difficult for them to understand more complex permutation and combination concepts. Students' skills in operating factorials relate to their understanding of the basic concept of factorials. Additionally, students' skills in performing multiplication calculations also represent an instrumental obstacle they face.

3.1.2 Didactical Obstacle

Based on the results of the analysis of ontogenic obstacles experienced by students, researchers suspect that students also experience didactical obstacles. Didactical obstacles are barriers in education that hinder students from understanding or learning a particular concept or topic. These obstacles can stem from various factors, such as an inadequate curriculum, inappropriate teaching methods, or difficulties students face in grasping complex concepts. Didactical obstacles can negatively impact students' ability to learn and achieve the desired academic performance. Therefore, teachers must identify and address didactical obstacles so that students can learn effectively and reach their full potential.

The first part of identifying the didactical obstacles faced by vocational high school students in solving counting rules problems is understanding students' opinions about their comprehension. Here is an excerpt from the interview conducted by the researcher and the students. To determine whether vocational high school students experience didactical obstacles, the researcher conducted interviews with students about how their teachers taught the counting rules material. The summary of the researcher's interviews with students is explained through several interview questions as follows:

Interviewer: Overall, do you understand the material on the counting rules?

Student 1: Not really, ma'am. Maybe because we learned online, but I don't understand well.

Student 2: I don't understand, ma'am. We learned entirely online, so the difficulty is that we couldn't ask the teacher directly.

Student 3: I don't understand, ma'am, because the lessons were just sent through files or PPT, and we were told to solve problems, but it wasn't explained again how to do it correctly.

Student 4: I don't understand well, ma'am, because I learned on my own. If the lessons were explained directly, maybe I would understand better.

Based on the interviews conducted with the students, all students admitted that they did not understand the counting rules material. This was due to ineffective learning methods and the lack of feedback provided after completing the given tasks. The students only studied from the learning resources provided or sent by the teacher and expressed that they would prefer if, after providing the material, the teacher explained it directly, whether online or offline (face-to-face).

Information on how the teacher conducted the learning of the counting rules was given by both students and the teacher. Students explained that the lessons were conducted using a WhatsApp group, with the teacher providing materials in the form of PPT slides and Word files. The teacher then gave practice problems and asked students to write their answers on paper or in a notebook, take photos of their work, and upload the tasks via a Google Form link provided by the teacher. The teacher also confirmed this method of teaching.

To document the teaching materials used by the teacher during the lessons, the researcher asked questions related to the teaching materials. Students reported that the teacher used Word files and PPT slides as teaching materials for the counting rules. A documentation study and analysis of didactical obstacles in the teaching materials used by the teacher were then conducted. Below is an excerpt from the teaching materials provided by the teacher.

Figure 4

Teacher's teaching material

ATURAN PENCACAHAN

Aturan Pencacahan meliputi 1. kaidah pncacahan|

2. Permutasi
3. Kombinasi

1. KAIDAH PENCACAHAN

Dapat diselesaikan dengan aturan penjumlahan, aturan perkalian yg di dalamnya adafaktorisasi. Hal ini yg kita bahas adalah faktorisasi, dengan rumus : $n! = 1.2.3.4.....n$

cth:

Hitunglah nilai dari a) $3!$

- b) $5! / 3!$
- c) $7!2! / 5!$

In teaching materials, the enumeration rules material begins by directly writing the factorial formula and providing example questions.

The teacher explained that when studying the counting rules (Figure 4), there are three subtopics: the Counting rules, Permutations, and Combinations. In the first part, titled The Counting Rules, the teacher directly explained that there are addition and multiplication rules, which include factorials. In the teaching materials, it was also evident that the teacher immediately wrote down the formula for factorials. The teacher then provided example problems to calculate factorial values and other factorial operations. This was similarly documented in the students' notebooks. Therefore, it can be concluded that in explaining the counting rules material, the teacher only introduced the concept of factorials and their operations without explaining the addition and multiplication rules, which are also part of the counting rules subtopic.

Figure 5

Discussion of Example Problems in Teaching Material

Hitunglah nilai dari a) $3!$

- b) $5! / 3!$
- c) $7!2! / 5!$

jawab: $3! = 1.2.3 = 6$

$5! / 3! = 1.2.3.4.5 / 1.2.3 = 4.5 = 20$ atau

$5! = 1.2.3.4.5 = 120$ jadi $5! / 3! = 120 / 6 = 20$

$7!2! / 5! = 1.2.3.4.5.6.7 . 1.2 / 1.2.3.4.5 = 6.7 . 1.2 = 84$ atau

$1.2.3.4.5.6.7 = 5040$, $1.2 = 2$ jadi $7!2! = 10080$

$1.2.3.4.5 = 120$ jadi $7!.2! / 5! = 10080 / 120 = 84$

Teaching materials are prepared without using proper notation, especially in writing factorial division operations

The next finding concerns the teaching materials, which were compiled with incorrect notation, particularly in writing division in factorial operations (Figure 5). In the teaching materials, the teacher used the notation “/” instead of the proper division notation. Furthermore, from the documentation study conducted by the researcher, the teaching materials for the subtopic of the counting rules did not include practice problems for students, especially those involving factorial operations.

Interviews were conducted with students to understand their opinions on which material was the most difficult to understand, particularly in the subtopics of the counting rules, permutations, and combinations. Below are some of the student responses and the questions posed by the researcher, as presented in Table 1.

Table 1
Student Responses Related to Difficult-to-Understand Material

Respondent	Statements
<i>Researcher</i>	<i>When studying the topic of counting rules which part is the most difficult to understand?</i>
<i>Student 1</i>	<i>I don't understand this material. I prefer or understand better if there is a teacher who explains it directly. If I study on my own from the material, I don't understand it well.</i>
<i>Student 2</i>	<i>The difficult part is when there are problems that are different from the examples, I get confused about whether to use permutations or combinations.</i>
<i>Student 3</i>	<i>Permutations are difficult, especially when there are problems that are different from the examples. The formulas also tend to get mixed up with combinations.</i>
<i>Student 4</i>	<i>The most difficult part is using the multiplication and addition rules. When working on problems, I get confused about the multiplication and addition rules at the end.</i>

Overall, students still find it difficult to understand the material on combinatorial rules, due to the design of the teaching materials provided, which are only procedural knowledge and have several shortcomings, including the sequence of material delivery that is not appropriate, the introduction of material that starts directly with the introduction of formulas, and the learning mode that is less conducive according to the students.

In addition to exploring information related to the didactic obstacles that arise in the learning of combinatorial rules, through interviews with students and a documentary study of teaching materials and student notebooks, the researcher also conducted interviews with teachers. The summary of the interview conducted with the teacher shows that the teacher admits that there is quite a difficulty in teaching, especially the material on permutations and combinations, with the mode of learning during the pandemic. The teacher explained that the combinatorial rule material is given at the end of the semester, so the teacher only introduces the factorial value, the permutation rule with different elements, and combinations. Furthermore, because the students do not have textbooks to take home, the teacher needs to prepare teaching materials to be distributed to the students. This results in the teaching materials being compiled directly by introducing the formula, example problems, and then practice problems. Based on this information, the researcher concludes that online learning during the pandemic, the limitations of learning resources owned by students, and the minimal learning time become factors of didactic obstacles experienced by students.

3.2 Discussion

3.2.1 Ontogenic Obstacle

The type of ontogenic obstacle that was also investigated is the instrumental ontogenic obstacle. Instrumental ontogenic obstacles refer to challenges that students face in learning or mastering a particular skill or activity. In the context of learning, instrumental ontogenic obstacles can occur when students do not have access to, or the ability to use, the tools or instruments needed to learn a particular skill or topic. These obstacles are related to the lack of access to tools or instruments necessary to complete a task or activity. According to Hasan (188: 2022), some of the ontogenic obstacles faced by vocational high school students in mathematics learning include (1) lack of student interest and readiness to receive learning; and (2) students having difficulty memorizing prerequisite material. Ontogenic obstacles arise in students, among other things, because students feel that learning mathematics is boring, and students feel that there is no new learning discovered when studying mathematics. Lack of motivation given by teachers can also result in students not being ready to learn (Lutfi et al., 2021). The research findings by Hasan align with the results of this study, which reveal that students experience instrumental ontogenic obstacles in performing basic multiplication operations and factorial operations. Permutation

and combination problems are closely related to multiplication operations, and students still face challenges in performing multiplication calculations, expanding factorial values, and operating factorial forms. This leads students to incorrect problem-solving solutions.

The Type 1 instrumental ontogenic obstacle experienced by students is related to their accuracy in performing multiplication operations. In solving counting rules problems, students' ability to perform multiplication operations is a fundamental instrumental skill. Therefore, this obstacle is considered an instrumental ontogenic obstacle faced by the students. The findings of this study align with research conducted by Dewi et al. (2022), which shows that ontogenic obstacles experienced by students include being less careful in performing addition and subtraction operations.

Instrumental ontogenic obstacle type 2 experienced by students involves using an incorrect form of the permutation formula. The correct permutation formula is $P_r^n = \frac{n!}{(n-r)!}$, but students use the incorrect formula $P_r^n = n!(n-r)!$. After confirming through interviews, students believe that $\frac{n!}{(n-r)!}$ can also be written as $n!(n-r)!$, leading them to use the latter form. This issue relates to students not being meticulous in applying the permutation formula correctly. Although students are aware of the permutation formula, they believe it can be expressed in different forms. The obstacle encountered by students in breaking down the permutation formula is an instrumental ontogenic obstacle experienced when solving counting rules. Combinatorial rule problems, including counting rules, are often challenging to categorize and solve with predictable algorithms; the ability to identify and utilize relationships between situations is crucial in solving counting rules problems (Lockwood, 2011). Errors in recognizing permutation formulas in this study are also observed in other research, where instrumental ontogenic obstacles occur when students mistakenly identify the guideline on images provided in material about triangles and quadrilaterals (Lutfi et al., 2021).

Instrumental ontogenic obstacle type 3 experienced by students involves their understanding of factorial notation. In Figure 3, it is shown that students decompose the notation $13! = 10! \times 3!$ even though these two expressions have different values/meanings. Students' understanding of factorial notation represents an instrumental ontogenic obstacle faced by vocational high school students. This finding aligns with Anggraini's (2022) research, which indicates that vocational high school students also struggle with factorial calculations. Therefore, ensuring that students understand factorial notation and perform operations with factorial expressions is a crucial learning trajectory for students in mastering counting rules (Jatmiko et al., 2017).

3.2.2 Didactical Obstacle

Several important findings regarding the design of teaching materials delivered by teachers include the following: First, teachers do not seem to fully provide knowledge related to counting rules. Teachers only explain factorials to students, while the rules of addition and multiplication are not explained in the teaching materials. Didactic obstacles greatly depend on the teacher's belief in preparing mathematical concepts and learning for their students (Sbaragli et al., 2011). In learning combinatorial rules, understanding the rules of addition and multiplication is very important because these two rules help us calculate the number of possible outcomes in various situations. In addition, the rules of addition and multiplication also form the basis for finding more complex combinatorial formulas, such as permutations and combinations. Therefore, explaining the rules of addition and multiplication is very important for understanding the concept of combinatorial rules as a whole. Didactical obstacles arise due to the limited presentation of prerequisite material (Jamilah et al., 2024).

The rule of addition is used when we want to calculate the total number of possibilities from two or more independent events. If there are n ways to do task A and m ways to do task B, then there are $n + m$ ways to do task A or task B. Example: If there are 3 routes to go to city A and 4 routes to go to city B, then there are $3 + 4 = 7$ routes that can be chosen to go to city A or city B. On the other hand, the rule of multiplication is used when we want to calculate the total number of possibilities from two or more dependent or sequential events. If there are n ways to do task A and m ways to do task B after task A is completed, then there are $n \times m$ ways to do tasks A and B sequentially. Example: If there are 3 routes to go

to city A and after that there are 4 routes to go to city B, then there are $3 \times 4 = 12$ ways to go to city B through city A.

In addition to not explaining the material on the rules of addition and multiplication, teachers also do not explain the rules of permutation with identical elements and cyclic permutations in the design of teaching materials used. Didactic obstacles occur when the learning process does not run smoothly, causing difficulties for students in understanding the material (Andani, Jamilah & Hartono, 2021). After the researcher confirmed with the teacher, the teacher conveyed that at that time, learning was done online, so learning was not effective. In addition, the combinatorial rule material was given at the end of the semester, so there was only time for a few meetings left. The teacher decided to choose the sub-material of the factorial and its operations, permutation with different elements, and combination and its applications. Therefore, the didactic design that will be compiled needs to present the combinatorial rule material completely.

The second finding is that in the teaching materials used, when explaining the concepts of permutation and combination, teachers always start the introduction of concepts by directly introducing formulas, not starting from discussing problems in everyday life or the world of work in the vocational field for vocational high school students. Learning on combinatorial rule material has not facilitated students to express their opinions, because learning focuses on the material delivered in the textbook (Permatasari et al., 2011). Students need to be trained to think and build knowledge independently, not by memorizing formulas directly. Without presenting relevant problems, students may have difficulty understanding the concepts of permutation and combination and relating them to real-life situations. This can lead to shallow understanding and difficulty in applying the concept to different problems. In addition, students may feel uninterested or unmotivated to learn combinatorial rules if they do not see the relevance or practical application of these concepts. Presenting interesting and contextual problems helps students see the importance of the topic in everyday life. Without presenting problems that encourage students to think and explore concepts, students may not develop good problem-solving skills. This ability is important for success in mathematics and many other fields. Presenting problems before introducing formulas allows students to try to find solutions in their own way, thus developing critical thinking skills. Without this opportunity, students may become too dependent on formulas and less able to think independently and creatively. In addition, introducing formulas without context or relevant problems can make it difficult for students to remember those formulas. Understanding the background and reasons behind formulas helps students remember and apply them more effectively.

The third finding concerns the use of mathematical notation used by teachers in explaining the division form in factorial operations. The use of a slash notation in writing division can cause problems or confusion for students, especially if the context or notation is not clear enough. Using a slash in expressions involving multiple operations can lead to errors in the order of operations if parentheses are not used correctly. This can confuse the priority of operations that need to be performed. In the context of algebra, especially when dealing with equations or functions, the use of a slash can make expressions appear ambiguous or unclear. In this case, it is better to use horizontal fraction notation.

Based on student responses, the researcher concludes that the difficulties experienced by students are related to how the design of teaching materials and learning activities are conducted. Students are still very dependent on teaching materials and the way teachers teach. Didactic obstacles arise due to inaccuracies in choosing the learning design, so students' understanding in this case is very dependent on the quality of the teacher in preparing the learning design (Sunariah & Mulyana, 2020; Sudirman et al., 2022).

The research results show that, overall, the materials are presented with a pattern of defining concepts, applying formulas, and providing example problems related to the use of formulas or rules. Students have not been facilitated in performing mental actions when facing the given problems. The materials present formulas directly, so students do not have the opportunity to build knowledge independently and have not been given space for discussion. Students are only provided with practice problems that are identical to the examples in the instructional materials. Other findings indicate that the materials are not presented using appropriate mathematical notation. For instance, the symbol "/" is used

to indicate division in factorial operations. Using correct notation in instructional materials is a key factor that can create learning obstacles for students. Counting rules material, such as addition and multiplication rules for events, is not covered in the materials. However, addition and multiplication rules are prerequisite knowledge needed by students to solve permutation and combination problems. Permutations are also not explained in full. The materials only cover permutations of distinct elements, whereas there are also topics on permutations of identical elements and cyclic permutations. A more detailed analysis of didactical obstacles is explained in the section on didactical obstacles.

The results of this study are in line with several research results related to didactic obstacles, including the following. Inaccuracies in delivering lesson material, the use of teaching methods that are not suitable for student needs, lack of detail in teaching materials, and inappropriate alignment in the learning flow are also factors that hinder the didactic process (Andani et al., 2021). Obstacles experienced by vocational high school students in solving counting problems include materials that have a high level of difficulty (Nugroho, 2021). Didactic obstacles are caused by materials in teaching materials that are not properly structured and there are conceptual errors in the teaching materials used, and teachers teach concepts that do not match the concepts that should be (Novianda, 2022).

4. Conclusion

During the COVID-19 pandemic, vocational high school students faced various challenges with online mathematics learning. The research revealed that students struggled with the instrumental skills needed to solve counting rule problems, such as obstacle with multiplication operations, permutation formulas, and factorial notation. Upon analyzing the ontogenic obstacles through interviews and document studies, it was found that students also faced didactical obstacles. Key concepts like the multiplicative and additive principles, which are essential for solving combinatorics problems, were not covered in the lessons. Overall, didactical obstacle stemmed from the design of instructional materials, content delivery, and the alignment of learning activities, which were not always conducive to effective learning. The study highlights the importance of ensuring that teachers consider students' readiness, especially in mastering prerequisite materials like permutation and combination. Additionally, the research underscores the need for well-structured instructional materials that follow the correct sequence of topics and use proper notation. Introducing the multiplicative and additive principles as foundational concepts for combinations is crucial. Instead of starting with formulas and exercises, instructional materials should begin with problem-solving to encourage mental engagement.

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Author Contribution

Tri Nopriana: Conceptualization, Writing – Original Draft, Editing and Visualization;
Sri Asnawati: Writing – Review & Editing, Formal analysis, and Methodology;
Tatang Herman: Validation and Supervision
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The authors declare no conflict of interest.

Additional Information:

Additional information is available for this paper.

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