

Students' Mathematical Reasoning: A Systematic Literature Review with a Focus on Metaphorical Thinking-Based Learning Approaches

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Abstract

Mathematical reasoning is a fundamental component of meaningful mathematics learning; however, research on instructional approaches that systematically support its development remains fragmented. In particular, studies on mathematical reasoning and metaphorical thinking have largely evolved independently, resulting in limited integrative insights. This systematic literature review (SLR) aims to synthesize research trends on students' mathematical reasoning over the past decade and to examine the role of metaphorical-thinking-based learning in supporting its development. Following the PRISMA 2020 framework, a systematic search of the Scopus and ERIC databases identified 36 peer-reviewed journal articles published between 2017 and 2025. These studies were analyzed using reflexive thematic analysis, focusing on research trends, instructional implementations, metaphor types, learning models, and reported effects on mathematical reasoning. The findings indicate a shift toward a multidimensional view of mathematical reasoning encompassing cognitive, representational, social, and contextual aspects. Metaphorical thinking predominantly functions as a cognitive-linguistic approach embedded within diverse instructional models. Conceptual and embodied metaphors are most prevalent, supporting reasoning through cognitive bridging, enriched mental representation, and increased engagement. Nevertheless, gaps remain regarding explicit interventions, longitudinal evidence, and comparative analyses of metaphor types.

Keywords: Mathematical reasoning, Metaphorical thinking, Mathematics education, Instructional models, Conceptual metaphors



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INTRODUCTION

Mathematical reasoning is a fundamental competency in mathematics education, as it enables students to explain relationships among concepts, justify solution strategies, and construct logical mathematical arguments. This perspective is consistent with the National Council of Teachers of Mathematics (NCTM), which positions reasoning and proof as a core process standard, emphasizing students' ability "to construct viable arguments and critique the reasoning of others." Jeannotte & Kieran (2017) further assert that reasoning forms the foundation of meaningful mathematical thinking.

In the Indonesian context, the need to strengthen mathematical reasoning is underscored by international assessment results. Reports from the Program for International Student Assessment (PISA) indicate that Indonesian students' mathematical literacy remains below the OECD average, particularly on tasks requiring higher-order reasoning, modeling, and non-routine problem solving. These findings suggest persistent difficulties in generalization, inference, and mathematical justification, which align with national studies reporting challenges in argument construction, strategy explanation, and representation coordination (Hansen & Naalsund, 2025; Siregar et al., 2025).

Over the past decade, numerous studies have explored instructional strategies to enhance students' mathematical reasoning, including reasoning-oriented task design (Cervantes-Barraza & Araujo, 2023),

technology- and web-based learning environments (Adelia et al., 2025), and pedagogical models emphasizing exploration, discussion, and active knowledge construction. Collectively, these studies indicate that reasoning develops most effectively when students are encouraged to formulate ideas, test conjectures, and reflect on their thinking.

Parallel to this line of research, metaphorical thinking has gained increasing attention as an approach to support students' understanding of abstract mathematical concepts. Metaphors allow learners to map abstract ideas onto concrete experiences, thereby enhancing conceptual understanding, problem solving, and mental representations. Research shows that metaphors operate not only through language but also through gestures, visualizations, and embodied cognition (Alcaraz-Carrión et al., 2022; Pansar, 2018), and can support conceptual mapping and meaning-making in mathematics (Purwasi & Fitriyana, 2021; Setiani et al., 2018).

Despite the growth of research on both mathematical reasoning and metaphorical thinking, these two strands have largely developed independently. Previous reviews tend to focus on metaphors in relation to conceptual understanding or general literacy rather than mathematical reasoning (Falani et al., 2023; Ghifari, 2025), while studies on reasoning rarely examine metaphors as cognitive tools for inference, argumentation, or conceptual structuring. Moreover, limited research has systematically examined different types of metaphors (conceptual, structural, orientational, and gestural), their relative effectiveness, or the instructional models that integrate metaphorical thinking to support reasoning. To date, no Systematic Literature Review (SLR) has comprehensively integrated research trends on mathematical reasoning with metaphorical thinking in mathematics education.

To address these gaps, this study conducts a Systematic Literature Review (SLR) focusing on the relationship between metaphorical thinking and mathematical reasoning in mathematics education. The review aims to synthesize existing evidence regarding the implementation of metaphorical thinking-based learning approaches, their role in supporting the development of students' mathematical reasoning, the dominant types of metaphors employed, and the instructional models integrated within such approaches. Specifically, this study examines research trends on students' mathematical reasoning in mathematics education over the past decade, explores how metaphorical thinking-based learning has been implemented, investigates the ways in which this approach influences or supports the development of mathematical reasoning, identifies the types of metaphors most commonly employed, and analyzes learning models that can be integrated with metaphorical thinking-based approaches and their contributions to students' reasoning development. Furthermore, the review seeks to identify existing research gaps in studies examining the relationship between metaphorical thinking-based learning and students' mathematical reasoning. Overall, this SLR is expected to contribute both theoretically and practically to the development of metaphor-based mathematics learning designs and to enrich the body of literature on students' mathematical reasoning.

Research Objectives:

The main objective of this Systematic Literature Review (SLR) is to synthesize and critically analyze research on students' mathematical reasoning with particular attention to the role of metaphorical thinking-based learning approaches in mathematics education. Specifically, this review aims to:

1. Identify and analyze research trends in students' mathematical reasoning in mathematics education over the past decade
2. Examine the implementation of metaphorical thinking-based learning approaches in mathematics education.
3. Analyze the influence of metaphorical thinking-based approaches on the development of students' mathematical reasoning.
4. Classify the types of metaphors commonly used in mathematics learning (e.g., conceptual, structural, orientational, embodied, and cultural metaphors) and their roles in supporting reasoning.

5. Identify learning models that can be integrated with metaphorical thinking-based approaches to enhance students' mathematical reasoning.
6. Identify research gaps and future directions in studies examining the relationship between metaphorical thinking-based learning and mathematical reasoning.

LITERATURE REVIEW

This study employs a Systematic Literature Review (SLR) to examine research trends on students' mathematical reasoning and metaphorical-thinking-based learning approaches in mathematics education over the past decade. The review was conducted following the PRISMA 2020 guidelines, which ensure transparency, rigor, and replicability in identifying, screening, and reporting relevant studies.

To synthesize findings from the selected literature, this study adopts a qualitative meta-synthesis approach using reflexive thematic analysis as proposed by Braun & Clarke (2022). This approach allows for the systematic identification of patterns, themes, and conceptual relationships across empirical studies originating from diverse educational contexts and methodological traditions.

The literature review focuses on mapping (1) research trends in mathematical reasoning, (2) the implementation of metaphorical-thinking-based learning approaches, (3) types of metaphors employed, (4) instructional models integrated with metaphorical thinking, and (5) reported impacts on students' mathematical reasoning. Through this synthesis, the review aims to provide a comprehensive and theoretically grounded understanding of how metaphorical thinking contributes to the development of mathematical reasoning.

RESEARCH METHOD

Research Design

This study adopts a Systematic Literature Review (SLR) design to investigate the relationship between metaphorical thinking and students' mathematical reasoning in mathematics education. The review process adheres to the PRISMA 2020 framework, which guides the stages of identification, screening, eligibility assessment, and inclusion of studies.

Data Sources and Search Strategy

The sample of this systematic literature review comprises journal articles examining mathematical reasoning and metaphorical-thinking-based learning in mathematics education. The literature search was conducted following the PRISMA 2020 guidelines to ensure methodological rigor and transparency.

Data collection involved a systematic search of two major databases, Scopus and ERIC. Keywords and search strings were developed through an exploratory theoretical search, incorporating core terms, related concepts, and synonyms to maximize coverage, as recommended by (Falani et al., 2023). The search terms included "mathematical reasoning," "reasoning ability," "mathematical thinking," "metaphorical thinking," "conceptual metaphor," and "mathematics education," supplemented by database-suggested terms and online thesauri. In line with previous studies, the development of keywords also utilized online thesauri, terminology from prior research, and automatically suggested keywords from databases such as Scopus and ERIC.

Using the Scopus and ERIC databases, the researchers conducted a literature search by applying the predetermined keywords and constructing a comprehensive search string. In Scopus, the search was performed using the TITLE-ABS-KEY field with the following search string: ("reasoning" OR "reasoning ability" OR "reasoning skill" OR "metaphorical thinking") AND ("mathematics education" OR "mathematics learning" OR "mathematics teaching" OR "learning mathematics").

Meanwhile, in ERIC, the search was conducted through the descriptor and abstract fields using the following search string: "metaphorical thinking" AND "mathematics education".

The selection of these two databases was based on recommendations from previous systematic studies, which emphasize that using more than one database is necessary to broaden the search coverage and reduce selection bias. Braun & Clarke (2022) note that multiple databases enhance the comprehensiveness of the search process. Scopus was selected because it is a globally recognized scientific citation database widely used in SLRs and meta-analyses, while ERIC is considered highly relevant for literature related to education and mathematics learning. From the initial search conducted across both databases, a total of 2,789 articles were obtained before entering the screening stage.

Inclusion and Exclusion Criteria

Inclusion and exclusion criteria were used as the basis for the screening process to ensure that only articles truly relevant to the research objectives were retained, as specified in Table 1 below.

Table 1. Inclusion and exclusion criteria

Criteria	Inclusion	Exclusion
Title and content	Meeting research requirements	Does not meet reasearch
Study limitations	Limited to studies published from 2017-2025	Studies published before 2017
Publication tipe	Journal articles	Conference proceedings, chapters in book, book series, book, etc.
Article field of study	Scratch in mathematics learning	Others
Language	English	Others
Access	Open access	Preview article or paid article

Screening Process (PRISMA Procedure)

The screening process followed a two-stage PRISMA procedure (see Figure 1).

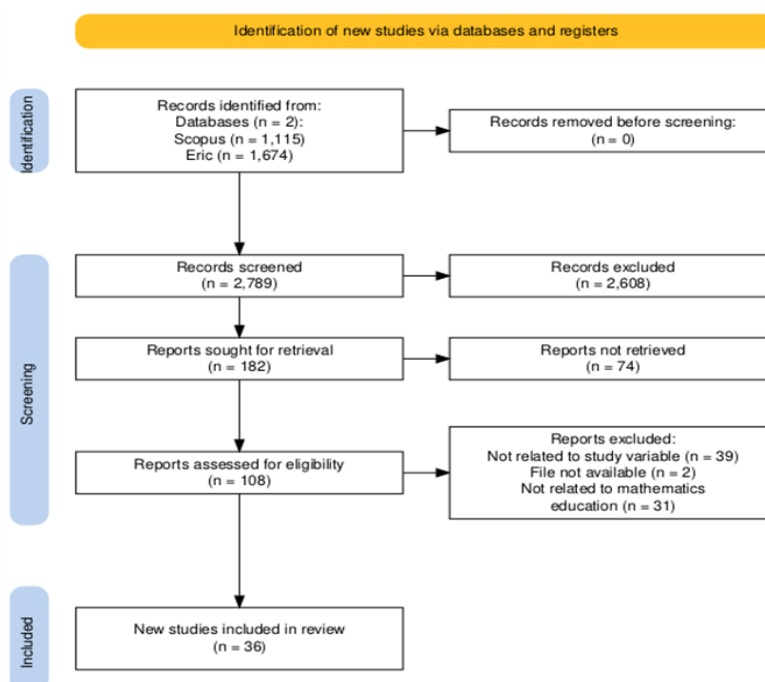


Figure 1. Articles screening process

Stage 1: Title–Abstract Screening

All 2,789 articles were screened based on relevance to mathematical reasoning and metaphorical thinking. Articles were excluded if they were non-empirical, outside mathematics education, not written in English, lacked full-text access, or were not journal articles. This stage excluded **2,608 articles**, leaving **182 articles**.

Stage 2: Full-Text Screening

A full-text review was conducted on the remaining articles. A total of **74 articles** were excluded due to language limitations, leaving **108 articles**. Further screening excluded articles irrelevant to the research focus (39), inaccessible articles (2), and studies outside mathematics education (31). Ultimately, **36 articles** met all inclusion criteria and were retained.

This rigorous screening process aligns with recommendations by Ghifari (2025), who emphasizes careful filtering due to conceptual overlap in metaphor-related research.

Quality Appraisal

A methodological quality assessment was conducted on the final 36 articles using the critical appraisal framework proposed by Connett (2018). A checklist was applied to evaluate methodological rigor, focusing on: (1) Relevance of research questions, (2) Clarity of study aims, (3) Appropriateness of research design, (4) Ethical considerations, (5) Rigor of data analysis, (6) Clarity of findings (7) Overall contribution of the study. Only studies meeting acceptable quality standards were included in the final synthesis.

Data Analysis

The 36 articles that passed the quality assessment were qualitatively analyzed using a reflexive thematic analysis approach to identify patterns, themes, and relationships across the findings. The analysis began with a comprehensive reading of each article, including the abstract, methodology, results, and discussion sections, to develop an in-depth understanding of the research context and focus. At this stage, all findings relevant to the research questions—particularly those related to mathematical reasoning and metaphorical thinking in mathematics education—were systematically extracted and summarized.

The next stage involved the initial coding process, which entailed identifying and labeling key units of information from each article. In accordance with the guidelines for reflexive thematic analysis proposed by Braun & Clarke (2022), the coding process was conducted in a flexible and iterative manner, involving the researcher's active interpretation of the data. The initial codes were then clustered based on recurring patterns or concepts identified across the included studies.

The generated codes were subsequently examined through a constant comparison process to ensure consistency and analytical depth. At this stage, findings across articles were compared to identify similarities, variations, contradictions, and potential conceptual relationships. This procedure aligns with the recommendations of Nowell et al., (2017), who emphasize the importance of iterative evaluation to enhance the credibility of thematic analysis results.

In developing the final themes and subthemes, the researchers synthesized meanings across diverse research contexts using a qualitative synthesis approach as described by Braun & Clarke (2022). This synthesis process involved integrating similar findings, exploring relationships among research variables (e.g., the relationship between metaphorical representations and the development of students' mathematical reasoning), and identifying the distinctive contributions of each article to construct a more comprehensive understanding.

After the final themes were established, the researchers constructed a synthesis narrative that comprehensively presented research trends, key patterns of findings, remaining research gaps, and the

conceptual contributions of each theme. This narrative subsequently formed the basis for the findings and discussion sections of this SLR.

FINDING AND DISCUSSION

Identifying and analyzing research trends in students' mathematical reasoning in mathematics education over the past decade.

Research on students' mathematical reasoning over the past decade has demonstrated increasingly robust and diverse developments. This trend is illustrated in Figure 2, which presents the publication trends in mathematical reasoning research from 2017 to 2025.

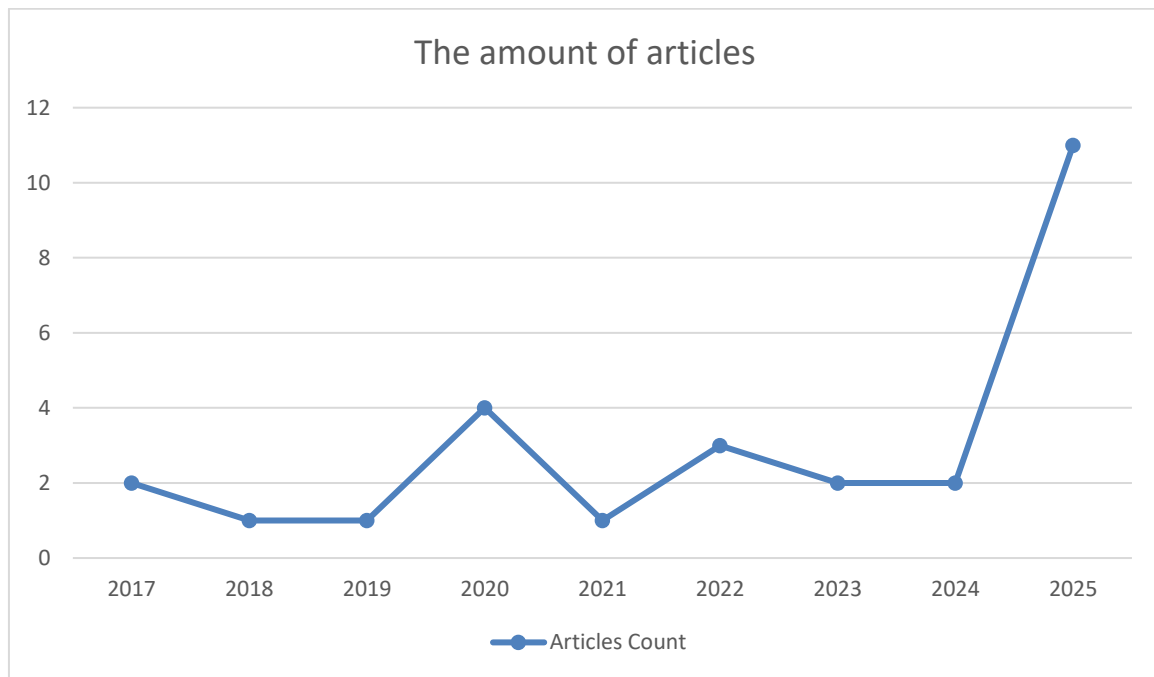


Figure 2. Publication trends in mathematical reasoning research

The data indicate that the popularity of research on mathematical reasoning has increased over the past decade. During the early period (2017–2019), the number of publications was relatively limited and tended to remain stable. Studies in this phase primarily focused on the development of conceptual frameworks of mathematical reasoning and on initial explorations of how students construct reasoning in contexts such as number, geometry, and mathematical generalization. Research by Schindler et al., (2017) and Jeannotte & Kieran (2017), for example, highlighted early attention to the theoretical and epistemological dimensions of mathematical reasoning.

A more pronounced increase became evident beginning in 2020, marked by a growing number of empirical studies examining mathematical reasoning through various instructional approaches, including technology-based learning, creative learning, and explorations of covariational reasoning. Although a fluctuation occurred in 2021, the publication trend rose consistently again during the 2022–2024 period. A particularly significant surge was observed in 2025, indicating that mathematical reasoning has become an increasingly central and relevant research issue in contemporary mathematics education.

During this later period, studies extended beyond focusing solely on students' mathematical reasoning outcomes to examine cognitive mechanisms, teachers' roles, digital learning environments, and the relationships between mathematical reasoning, self-regulated learning, and 21st century competencies.

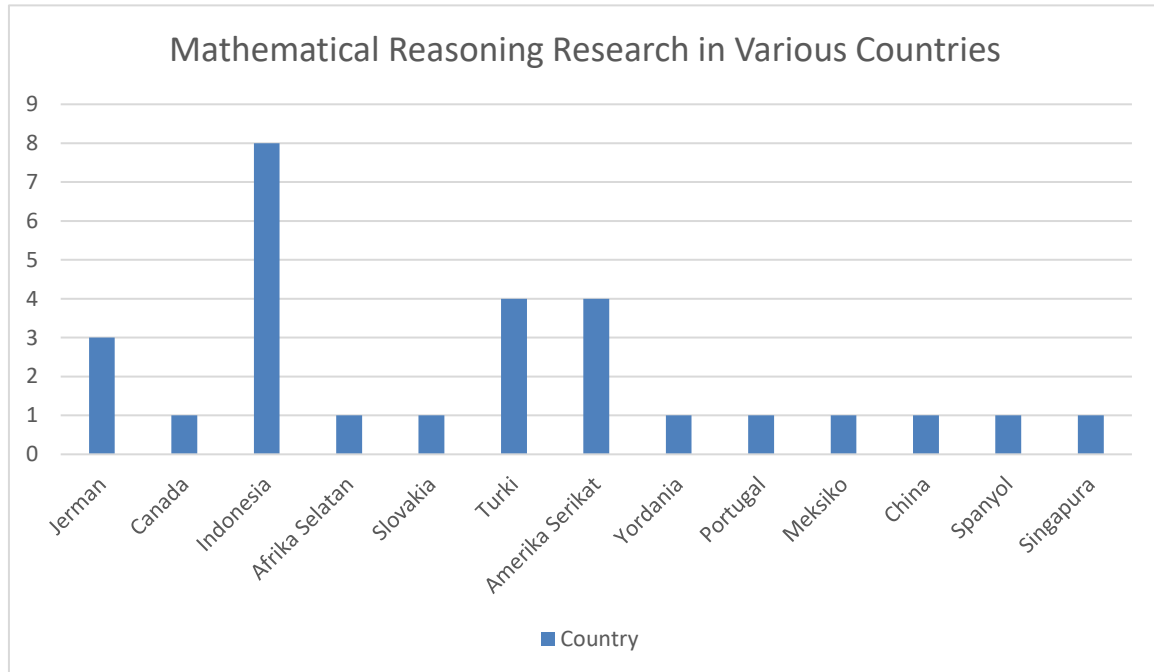


Figure 3. Geographical distribution of studies on mathematical reasoning in mathematics education

Mathematical reasoning is now viewed as a complex cognitive process associated with proof, conceptual understanding, problem solving, generalization, and mathematical representation. One notable trend is the growing attention to the development of mathematical reasoning through authentic learning environments, particularly in Indonesia. As shown in the Figure 3, Indonesia emerges as the largest contributor, reflecting the increasing focus of national researchers on mathematical reasoning as a core competence in mathematics curricula and instruction.

Studies from Indonesia generally emphasize the effects of instructional models, the development of learning environments, and the relationships between mathematical reasoning, self-regulated learning, and numeracy. Significant contributions are also evident from other countries, such as Turkey, the United States, and several European nations, which enrich the field through diverse theoretical and empirical perspectives.

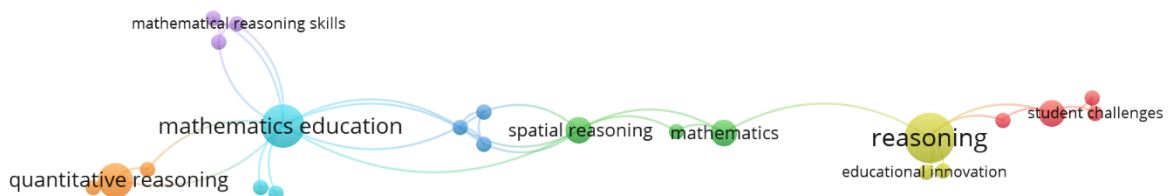


Figure 4. Network visualization of Research about mathematical reasoning

Figure 4 illustrates the relationships among various terms in research on mathematical reasoning. The interconnected network shown in the figure reveals clusters of terms that frequently co-occur, reflecting the main foci and directions of research over the past decade. The term “*reasoning*” appears as a central node with strong connections to “*educational innovation*” and “*student challenges*”. This pattern suggests that mathematical reasoning is conceptualized not only as a cognitive ability but also as a competence closely linked to instructional innovation and students’ learning challenges.

Another prominent cluster highlights the close association among the terms “*mathematics education*,” “*mathematics*,” and “*spatial reasoning*,” indicating that research on mathematical reasoning is largely situated within formal educational contexts, with particular attention to spatial reasoning. This finding aligns with the study by Fraihat et al., (2022), which demonstrated that situated learning environments can enhance students’ reasoning and proof abilities. The presence of the term “*mathematical reasoning skills*” linked to “*mathematics education*” further underscores that the development of reasoning skills remains a central objective in mathematics instruction.

Meanwhile, “*quantitative reasoning*” appears as a connected yet relatively distinct cluster, suggesting that although quantitative reasoning is related to mathematics education, it is often treated as a separate focus from general mathematical reasoning. This pattern reflects a conceptual and terminological differentiation within the literature.

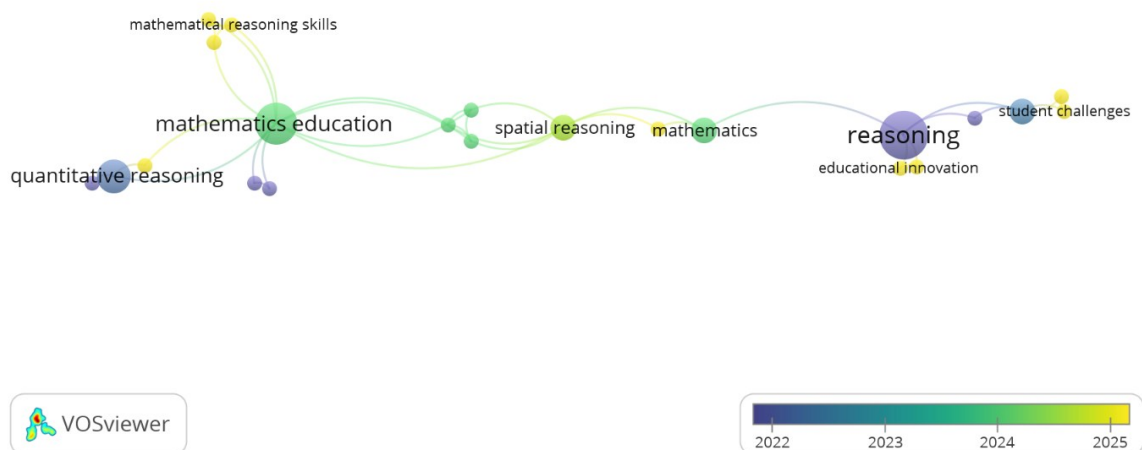


Figure 5. Overlay visualization of Research about mathematical reasoning

Figure 5 presents an overlay visualization of the keyword network in mathematical reasoning research. The colors in this visualization represent the temporal development of research foci, ranging from earlier studies to more recent investigations. The terms “*quantitative reasoning*” and “*mathematics education*” are predominantly displayed in darker colors, indicating that these topics were emphasized in earlier periods. In contrast, the terms “*reasoning*,” “*educational innovation*,” and “*student challenges*” appear in lighter colors, signaling an increased research focus on pedagogical innovation and students’ learning challenges in more recent years.

The literature positions “*spatial reasoning*” and “*mathematical reasoning skills*” as transitional constructs that bridge early conceptually oriented studies and more recent applied and contextually grounded research. Alongside this shift, research increasingly extends toward cultural, social, and pedagogical dimensions of mathematical reasoning. Cervantes-Barraza & Araujo (2023) show that cultural contexts enrich students’

engagement in reasoning, while studies such as Hansen & Naalsund (2025) conceptualize reasoning as a collaborative social practice shaped by interaction patterns and classroom dynamics.

Within specific reasoning domains, growing attention is evident in spatial, covariational, and visually mediated reasoning. Hasanah et al., (2024) identify spatial reasoning as central to geometry learning and cognitive development, whereas Kertil (2020) emphasizes covariational reasoning for understanding dynamic change through digital representations.

Research also highlights a strong link between mathematical reasoning and conceptual understanding. Studies by Bakar et al., (2018) and Schindler et al., (2017) associate weak reasoning with fundamental misconceptions, while Hamidah et al., (2025) report variations in reasoning ability among prospective teachers, underscoring the importance of early and systematic development in teacher education.

Methodologically, recent studies increasingly employ mixed methods, case studies, design-based research, and lesson study to capture reasoning processes in depth. Martins et al., (2023) demonstrate that lesson study, in particular, supports prospective teachers in developing instructional practices that foster students' mathematical reasoning.

Examining the implementation of metaphorical thinking-based learning approaches in mathematics education.

The results of the Systematic Literature Review (SLR) indicate that metaphorical thinking-based instructional approaches have been implemented in various forms and for diverse purposes within mathematics education. This approach aims to support students' understanding of abstract concepts through conceptual mapping from concrete experiences. In general, the implementation of this approach focuses on the use of different types of metaphors, including (1) conceptual metaphors, (2) structural metaphors, and (3) orientational metaphors. These metaphors function as cognitive tools that facilitate students' thinking processes in constructing meaning from mathematical concepts.

Several studies have reported the application of metaphorical thinking in the design of mathematical tasks that encourage students to engage in analogical mapping between everyday situations and mathematical structures. For instance, Purwasi & Fitriyana (2021) employed metaphor-based activities to enhance students' reasoning and problem-solving abilities. In this approach, students were guided to understand mathematical concepts through carefully designed visual and verbal analogies embedded in the learning process. Similarly, Alcaraz-Carrión et al., (2022) demonstrated that spontaneous gestures in the form of hand-movement metaphors can function as cognitive bridges in representing arithmetic operations.

In the context of classroom pedagogy, the teacher's role is crucial in explicitly employing metaphors as instructional strategies. Scheiner et al., (2022) emphasize that the use of metaphors in teachers' explanations not only helps students grasp challenging concepts but also influences how students organize and structure their mathematical experiences. In addition, Falani et al., (2023) report that metaphors play a significant role in conceptual representation, mathematical communication, and situational meaning-making.

Metaphor-based approaches have also been implemented in technology-enhanced learning and digital environments. Several studies have integrated metaphors into learning interfaces, virtual object manipulation, and the visualization of mathematical concepts. Findings reported by Ghifari (2025) indicate that mathematics learning becomes more effective when metaphors are embedded within digital instructional design.

Analyzing the influence of metaphorical thinking-based approaches on the development of students' mathematical reasoning.

Based on the synthesis of the selected studies, metaphorical thinking-based instructional approaches make a substantial contribution to the development of students' mathematical reasoning through several

interrelated mechanisms. First, metaphors function as a cognitive bridge between abstract mathematical concepts and students' concrete, familiar experiences. This bridging process supports the construction of more logical and coherent reasoning structures, enabling students to connect mathematical ideas and justify problem-solving steps more effectively (Purwasi & Fitriyana, 2021). Empirical evidence further confirms that metaphor-based interventions significantly enhance students' mathematical reasoning compared to conventional instruction (Pantaleon et al., 2024), while also strengthening mathematical connections through more structured conceptual representations (Putri et al., 2022).

Second, metaphorical thinking supports the development of richer mental representations. By mapping mathematical structures onto visual, gestural, or experiential domains, metaphors help students visualize quantitative relationships and underlying structures. For instance, Alcaraz-Carrión et al., (2022) demonstrate that metaphors expressed through co-speech gestures enable students to construct more robust mental representations, thereby strengthening their capacity for inference and generalization. Similar findings are reported in studies examining the use of metaphors to support visualization and understanding in contexts such as percentage operations (Giberti et al., 2023), and trigonometric word problems, where metaphor use enhances students' visualization skills and deepens their understanding of mathematical relationships (Kursius et al., 2025).

Third, metaphor-based approaches promote cognitive engagement by encouraging students to explore patterns, construct analogies, and develop personal interpretations of mathematical concepts. Classroom-based evidence indicates that the explicit use of metaphors in discussion and instruction stimulates higher-order thinking processes, including justification, critique of solutions, and the construction of mathematical arguments (Hendriana et al., 2017). This engagement aligns with broader findings showing that metaphors are not employed as isolated instructional techniques but are integrated across diverse instructional models, representational practices, and authentic learning activities (Putri et al., 2022; Scheiner et al., 2022).

Overall, the accumulated evidence suggests that metaphorical thinking enriches students' learning experiences by simultaneously strengthening conceptual understanding, mental representation, and cognitive engagement, which constitute core components of mathematical reasoning. The flexibility of metaphor use across instructional contexts highlights its potential as a powerful pedagogical strategy. Consequently, integrating metaphor-based approaches into mathematics curricula is increasingly relevant for fostering deeper, more meaningful, and contextually grounded mathematical understanding among students.

Classifying the types of metaphors commonly used in mathematics learning (e.g., conceptual, structural, orientational, embodied, and cultural metaphors) and their roles in supporting reasoning.

The findings of this systematic literature review demonstrate that metaphors play a central role in mathematics learning by supporting students' understanding of abstract concepts that are often difficult to access through formal symbolism alone. Metaphorical thinking functions as a cognitive mechanism that enables learners to map concrete experiences onto abstract mathematical ideas, thereby strengthening conceptual frameworks and mathematical reasoning (Scheiner et al., 2022).

Across the reviewed studies, several types of metaphors are identified. Conceptual metaphors are the most prevalent, commonly used to explain abstract ideas through familiar structures such as "mathematics as a journey," "numbers as objects," and "functions as machines," facilitating students' transition from intuitive to formal reasoning across educational levels (Falani et al., 2023; Purwasi & Fitriyana, 2021). In addition, recent research increasingly highlights the role of gestural or embodied metaphors, which employ bodily movements and gestures to enhance visualization and reasoning, aligning with principles of embodied mathematical cognition (Alcaraz-Carrión et al., 2022).

Structural metaphors contribute to the development of relational and deductive reasoning, particularly in advanced topics. Empirical evidence shows that structural analogies support students' understanding of abstract algebra and interconceptual relationships in functions, geometry, and algebra (Hicks, 2024; Pantaleon et al., 2024). Although less frequently discussed, orientational metaphors facilitate understanding through spatial representations, such as number lines (Schindler et al., 2017), while cultural or ethnomathematical metaphors connect mathematical ideas to students' lived experiences, promoting contextualized reasoning (Cervantes-Barraza & Araujo, 2023)

Despite their overall effectiveness, the impact of metaphors is highly context-dependent. No single metaphor type is universally effective; instead, their success depends on instructional design, learning objectives, and mathematical content. Conceptual metaphors are particularly effective for introducing foundational ideas, whereas structural metaphors better support relational understanding, underscoring the importance of systematic and contextually relevant implementation (Hasnarika, 2022; Wahid et al., 2024). Overall, metaphors function not merely as illustrative devices but as powerful pedagogical tools that support higher-order mathematical thinking, while further research is needed to refine their integration into instructional models.

Identifying learning models that can be integrated with metaphorical thinking-based approaches to enhance students' mathematical reasoning.

The literature review indicates that metaphorical thinking in mathematics education is not applied as an isolated approach but is commonly integrated with various instructional models to support meaning-making, conceptual understanding, and mathematical reasoning. Across the reviewed studies, metaphorical thinking is embedded—explicitly or implicitly—through analogies, conceptual metaphors, and embodied representations. Notably, Realistic Mathematics Education (RME) aligns closely with metaphorical thinking by emphasizing real-life contexts and meaningful representations (Adelia et al., 2025). Similarly, ethnomathematics integrates conceptual and structural metaphors through culturally grounded tasks that enrich students' reasoning (Cervantes-Barraza & Araujo, 2023). Embodied learning models highlight gestures as “metaphorical bridges” that support arithmetic understanding (Alcaraz-Carrión et al., 2022). In addition, Problem-Based and Task-Based Learning models frequently involve metaphor-based meaning construction in open and investigative tasks (Masfingatini et al., 2020; Medová et al., 2020). Metaphor-based approaches are also evident in early childhood conceptual learning models such as Conceptual PlayWorld (Zhang et al., 2025) and in technology-enhanced learning environments that employ visual metaphors to support covariational reasoning (Kertil, 2020).

Identifying research gaps and future directions in studies examining the relationship between metaphorical thinking-based learning and mathematical reasoning.

Based on the analysis of 36 selected studies, the findings indicate that research on metaphorical thinking and mathematical reasoning has largely developed in parallel, with limited empirical studies explicitly integrating metaphorical thinking as a primary instructional intervention for enhancing mathematical reasoning. Although some studies highlight the potential of metaphor use in mathematics learning, metaphorical thinking is rarely conceptualized as a central cognitive strategy within reasoning-focused research. For example, while studies by Purwasi & Fitriyana (2021) and Setiani et al., (2018) highlight the influence of metaphors in mathematics learning, research on mathematical reasoning, such as that conducted by Fraihat et al., (2022) and Hansen & Naalsund (2025) seldom addresses metaphors as deliberate cognitive strategies for enhancing students' reasoning.

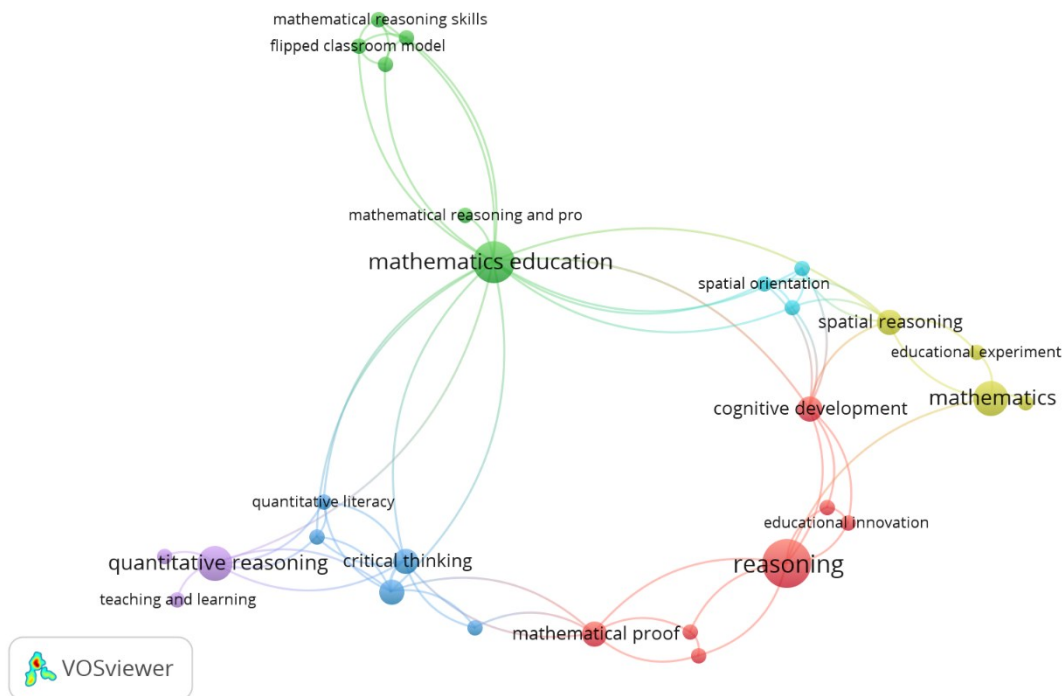


Figure 6. Network visualization of relationship between metaphorical thinking-based learning and students' mathematical reasoning

As illustrated in Figure 6, the keyword co-occurrence network is dominated by general themes such as “reasoning,” “mathematics education,” and “mathematical reasoning skills,” while metaphorical thinking does not emerge as a prominent keyword nor form a distinct research cluster. Instructional approaches appear as relatively small and weakly connected nodes, indicating weak integration between specific learning approaches and studies on mathematical reasoning.

In addition, the reviewed literature reveals limited investigation into the types and instructional functions of metaphors used in mathematics learning. Many studies address metaphors implicitly, such as through gestures, contextual representations, or cultural metaphors, without systematically examining how different forms of metaphorical thinking contribute to specific dimensions of mathematical reasoning. Furthermore, most existing studies employ cross-sectional designs or short-term interventions, despite the gradual nature of both metaphorical thinking development and mathematical reasoning, resulting in a lack of longitudinal evidence.

Disparities across educational levels also represent a significant gap. Metaphorical thinking-based learning is predominantly examined at the elementary and secondary school levels, whereas mathematical reasoning is more frequently studied among pre-service teachers or university students. This imbalance limits a comprehensive understanding of how metaphorical thinking supports reasoning across different stages of cognitive development.

Moreover, the integration of interdisciplinary theoretical frameworks remains insufficient. Although theories such as Conceptual Metaphor Theory and Embodied Cognition are occasionally referenced, most studies rely on isolated theoretical perspectives and do not explicitly connect these theories with established models of mathematical reasoning. Finally, the review highlights a lack of technology-enhanced studies that integrate metaphorical thinking-based learning. While digital tools and dynamic learning environments have

been shown to support mathematical reasoning, their potential role in facilitating metaphorical thinking and strengthening reasoning processes remains largely unexplored.

Taken together, the convergence of systematic content analysis and VOSviewer-based bibliometric evidence demonstrates a substantial research gap in the literature. Specifically, there is a lack of comprehensive synthesis that examines how metaphorical thinking-based learning is theoretically grounded, empirically implemented, and methodologically evaluated in relation to students' mathematical reasoning.

CONCLUSION

The findings indicate that research trends over the past decade have conceptualized mathematical reasoning as increasingly multidimensional, encompassing cognitive, social, representational, and affective aspects. The results also reveal a growing emphasis on contextualized mathematical reasoning through the integration of cultural contexts and authentic tasks, which enrich students' reasoning processes. The use of a metaphorical thinking approach not only facilitates students' understanding of mathematical concepts but also contributes to the development of their critical thinking and creativity. Overall, the findings demonstrate that metaphorical thinking enhances students' learning experiences while strengthening conceptual understanding, mental representations, and cognitive engagement, which constitute the core components of mathematical reasoning. The conclusion contains the main points of the article. Conclusions should not repeat what has been written in the Results and Discussion section, but discuss important results, applications and developments of the research carried out. This section should also be able to show whether the research objectives can be achieved.

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