CALENDAR VARIATION MODEL FOR FORECASTING TIME SERIES DATA WITH ISLAMIC CALENDAR EFFECT

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ABSTRACT

The aim of this paper is to develop a statistical model for explaining and forecasting the time series that contains Islamic Calendar effect. In time series literature, calendar variation is defined as a periodic and recurrent pattern with variation length period that usually caused by cultures and religions of people in a certain area. In Indonesia, the effect of the Eids holiday in many daily activities, such as transportation, inflation and consumption, is one example of calendar variations. This holiday happens on different month after three years or shift to previous month after at the same month on three years. This paper evaluates the disadvantage of seasonal classical time series model, such as Winter's, Decomposition and ARIMA models, and develops a Calendar Variation model for forecasting time series that contain Islamic Calendar Effect. In this research, a real data about monthly sales of sardines are used as a case study. The results show that classical time series models, such as Winter's, Decomposition and ARIMA models, cannot describe the calendar variation effect and yield invalid and unreliable forecast, particularly at the time (month) when the calendar variation happens. On the contrary, Calendar Variation model is a model that can explain precisely the impact of the calendar variation effect and gives valid and reliable forecasts.

Keywords: Calendar Variation model, Islamic Calendar, time series

Many business and economic time series are non-stationary time series that contain trend, seasonal variations and cycles. The trend is the long-term component in the time series that represents the growth or decline over an extended period of time. Seasonality is a periodic and recurrent pattern caused by factors such as weather, holidays, or repeating promotions. Another important pattern seen as seasonality is calendar variation. The calendar variation is also a periodic and recurrent pattern with variation length period. It is usually caused by cultures and religions of people in a certain area.

In Indonesia, the effect of the Eids holidays in many daily activities, such as transportation, inflation and consumption, is one example of calendar variations. This holiday happens on different month after three years or shift to previous month after at the same month on three years. Accurate forecasting of trend, seasonal and calendar variation in time series is very important for effective decisions in retail, marketing, production, inventory control, personnel, and many other business sectors [9]. Thus, how to model and forecast time series containing calendar effect has long been a major research topic that has significant practical implications.

There are some forecasting techniques that usually used to forecast time series with trend and seasonality, including additive and multiplicative methods. Those methods are Winter's exponential smoothing, Decomposition and Autoregressive Integrated Moving Average (ARIMA) models (see e.g. [3, 8]). Recently, Neural Networks (NN) models are also used for time series forecasting (see e.g. [7, 11, 12]).

The effect of calendar variation to the forecasting result at a time series has been discussed in many papers (see e.g. [1, 2, 10, 12, 13]). Sullivan et al. [13] studied the effect of calendar variations in stock returns. Bokil and Schimmelpfennig [2] investigated the effect of calendar variation to inflation forecasting in Pakistan. Whereas, Suhartono et al. [12] studied the calendar variation effect to inflation forecasting in Indonesia.

The purpose of this paper is to evaluate the disadvantage of seasonal classical time series model, such as Winter's, Decomposition and ARIMA models, and to develop a Calendar Variation model which is appropriate for forecasting time series that contain Islamic Calendar Effect.

SEASONAL TIME SERIES MODELS

In this section, the author will give a brief review of these forecasting models that usually used for forecasting a seasonal time series, particularly additive seasonal models.

Winter's Exponential Smoothing Model

Exponential smoothing is a procedure for continually revising an estimate in the light of more recent experiences. Winter's model is exponential smoothing model that is usually used for forecasting trend and seasonal time series. The four equations used in additive Winter's model are as follows (see [8]):

(i). The exponentially smoothed series:

$$L_t = \alpha(y_t - S_{t-L}) + (1-\alpha)(L_{t-1} + T_{t-1}) \qquad (1)$$

(ii). The trend estimate:

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1} \qquad (2)$$

(iii). The seasonality estimate:

$$S_t = \gamma(Y_t - L_t) + (1-\gamma)S_{t-L} \qquad (3)$$

(iv). Forecast p periods into the future:

$$\hat{y}_{t+p} = L_t + pT_t + S_{t-L+p} \qquad (4)$$

where

α = smoothing constant $(0 < \alpha < 1)$
β = smoothing constant for trend estimate $(0 < \beta < 1)$
γ = smoothing constant for seasonality estimate $(0 < \gamma < 1)$
L = length of seasonality.

Decomposition Model

In this section the additive decomposition model will be presented. The key assumption inherent in this model is that seasonality can be separated from other components of the series. The multiplicative decomposition model is (see [3])

$$y_t = T_t + S_t + C_t + I_t \qquad (5)$$

where

y_t = the observed value of the time series in time period t
T_t = the trend component in time period t
S_t = the seasonal component in time period t
C_t = the cyclical component in time period t
I_t = the irregular component in time period t.

Seasonal ARIMA Model

The seasonal ARIMA model belongs to a family of flexible linear time series models that can be used to model many different types of seasonal as well as nonseasonal time series. The seasonal ARIMA model can be expressed as (see e.g. [5, 6, 14]):

$$\phi_p(B)\Phi_P(B^S)(1-B)^d(1-B^S)^D y_t = \theta_q(B)\Theta_Q(B^S)\varepsilon_t \ , \qquad (6)$$

where

$$\phi_p(B) = 1-\phi_1 B-\phi_2 B^2-\ldots-\phi_p B^p$$
$$\Phi_P(B^S) = 1-\Phi_1 B^S-\Phi_2 B^{2S}-\ldots-\Phi_P B^{PS}$$
$$\theta_q(B) = 1-\theta_1 B-\theta_2 B^2-\ldots-\theta_q B^q$$
$$\Theta_Q(B^S) = 1-\Theta_1 B^S-\Theta_2 B^{2S}-\ldots-\Theta_Q B^{QS},$$

where S is the seasonal length, B is the back shift operator and ε_t is a sequence of white noises with zero mean and constant variance. Box and Jenkins [4] proposed a set of effective model building strategies for seasonal ARIMA based on the autocorrelation structures in a time series.

CALENDAR VARIATION MODEL

Calendar variation effects model was originally introduced by Bell and Hillmer [1]. In general, the calendar variation model proposed in this paper is different from Bell and Hillmer model, i.e.

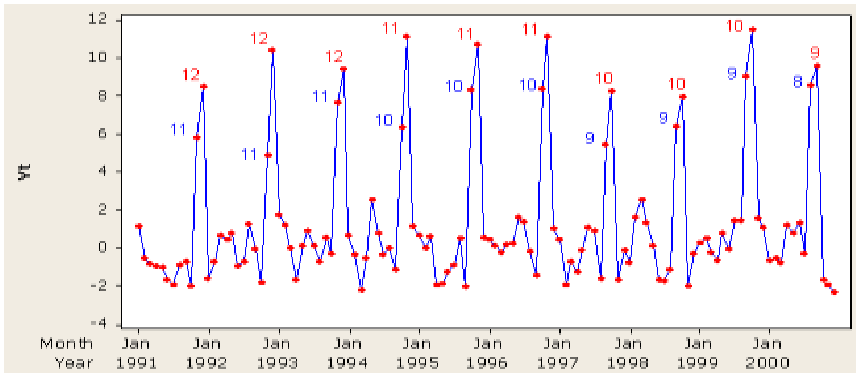
$$y_t = \omega_s(B)B^b CV_t + \frac{\theta_q(B)\Theta_Q(B^S)}{\phi_p(B)\Phi_P(B^S)} a_t, \qquad (7)$$

where

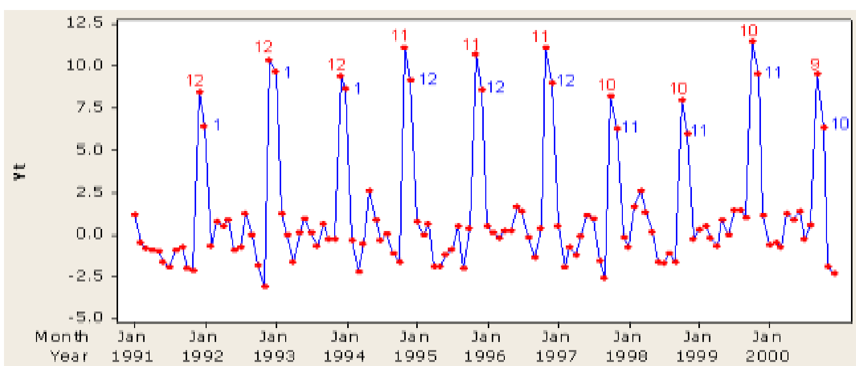
$\omega_s(B)$ = $\omega_0-\omega_1 B-\omega_2 B^2-\ldots-\omega_s B^s$,
b = time when the calendar variation effect happens,
CV_t = calendar variation variable, i.e. dummy variable which value is 1 for time at calendar variation and 0 for others.

An example of calendar variations that will be discussed in this paper is the Eids holiday. This holiday happens on different month after three years or shifts to the previous month after at the same month. It is caused the determination of this holiday based on the Islamic calendar, and it is different with Gregorian (Western) calendar that usually used in our daily life. In this case, CV_t has value 1 for month when this holiday happens and 0 for others.

In general, the form of this Calendar Variation model is the same as transfer function and intervention models (see [14]). The main different is the value *b* in this model does not have to be 0 or positive. The negative value of *b* indicates the effect of the calendar variation happens before the calendar variation period, 0 means that the effect happens at the same time, and positive indicates the delay effect of calendar variation. Examples of time series data followed calendar variation model can be seen in Figure 1.



(a). $Y_t = (8+10B)\, B^{-1} CV_t + \dfrac{(1+0{,}25B)}{(1-0{,}4B)}\, a_t$



(b). $Y_t = (10+8B)\, CV_t + \dfrac{(1+0{,}25B)}{(1-0{,}4B)}\, a_t$

Figure 1. Simulated Data Containing Calendar Variation Effects

RESEARCH METHODOLOGY

The purpose of this research is to study further and develop a time series model for explaining the effect of calendar variation and yielding forecast accurately of time series data containing calendar variation, i.e. the Eids holiday. Firstly, the disadvantages of seasonal classical time series model, such as Winter's, Decomposition and ARIMA models will be assessed. Finally, study to evaluate the forecast accuracy and diagnostic check of the appropriate model will be compared.

In this research, a real data about monthly sales of sardine, Kikku 155 gr, at PT. Blambangan Jaya, Banyuwangi, Indonesia are used as a case study. This data contains 64 observations, starting from January 1999 until June 2004. Figure 2 shows the time series plot of this monthly sales data.

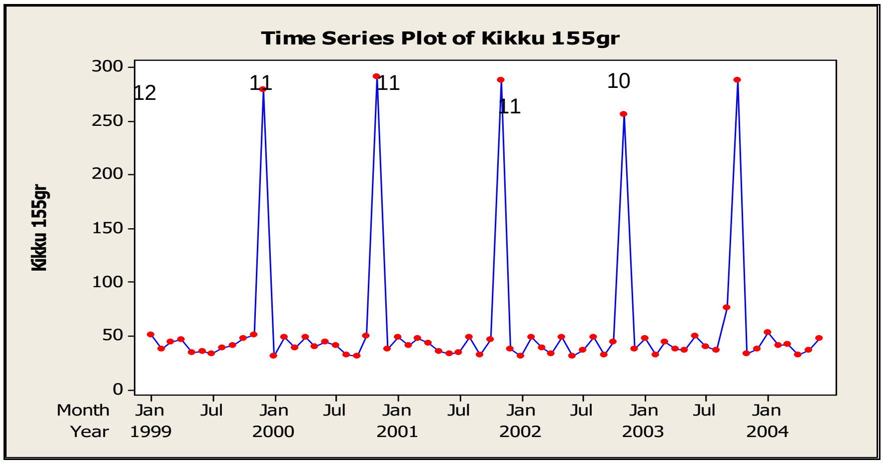


Figure 2. Time series plot of monthly sales Kikku 155 gr (in thousand cans)

Based on this plot, we can see clearly that monthly sales of this sardine have stationary pattern and only increase significantly at the month when the Eids holiday happens. We can also see that this holiday shifts to the previous month after three years. Rudely observation using graphical analysis usually gives conclusion that a time series contains seasonality pattern with 12 months period. Therefore, firstly the seasonal classical time series model, such as Winter's, Decomposition and ARIMA models for forecasting this data was applied.

EMPIRICAL RESULTS

In this section the empirical results for forecasting monthly sales sardine Kikku 155 gr by using Winter's, Decomposition, ARIMA and Calendar Variation models are presented and discussed. As a final point, the comparison of the forecast accuracy among these models is presented.

Result of Winter's Model

Graphical identification shows that data look as if it contains seasonality pattern with no variation. The result of additive Winter's model with α = 0,1; β = 0,2 and γ = 0,6 can be seen at Figure 3.

This result shows that Winter's model yields invalid forecasts particularly at the time (month) when the Eids holiday shifts to different month, i.e. at the first, second and fifth years. The forecasts at the second year show that the greatest forecast happens on December, as it happened at the first year. This condition is different with the actual data, i.e. the greatest sales at the second year happens on November. The same thing also occurs at the fifth year when the greatest forecast happens on November, and it is different with the actual, i.e. the greatest sales is on October.

The forecasts on the next six months at the sixth year show that Winter's model yields appropriate forecasts, i.e. the greatest sales forecast will occur on October as it happened at the fifth year.

Results of Decomposition Model

Figure 4 is the graphical result of the forecasting by implementing additive Decomposition model. Based on this figure, it can be seen clearly that Decomposition model yields invalid forecasts as Winter's model, i.e. particularly at the month when the Eids holiday shifts to different month.

Decomposition model yields invalid forecasts not only at the second year observations but also at the fifth, sixth and so on. It is caused this model assumes the greatest increasing sales happens continuously on November. From Figure 4, it can be observed clearly that the greatest sales forecasts on the next six months is at the sixth year or in 2004 it will happen on November. This condition is contrary to the actual that the Eids holiday, when the greatest monthly sales always happen, in 2004 will occur on October.

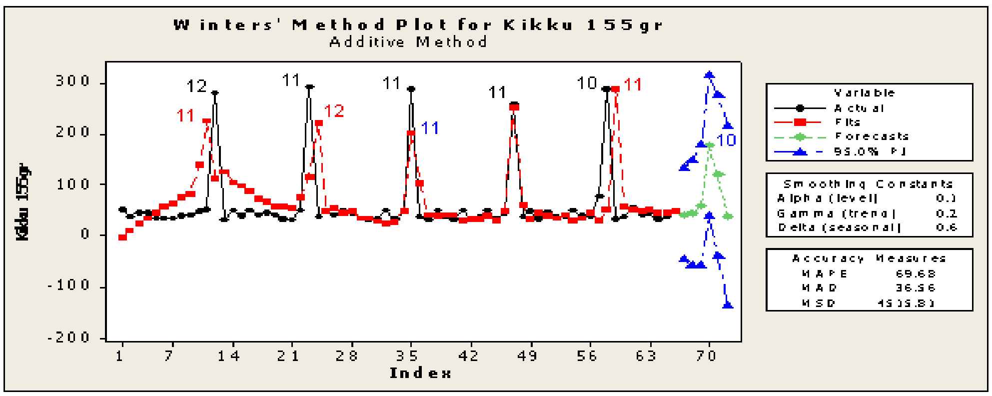


Figure 3. Forecasting results by using the Additive Winter's Model

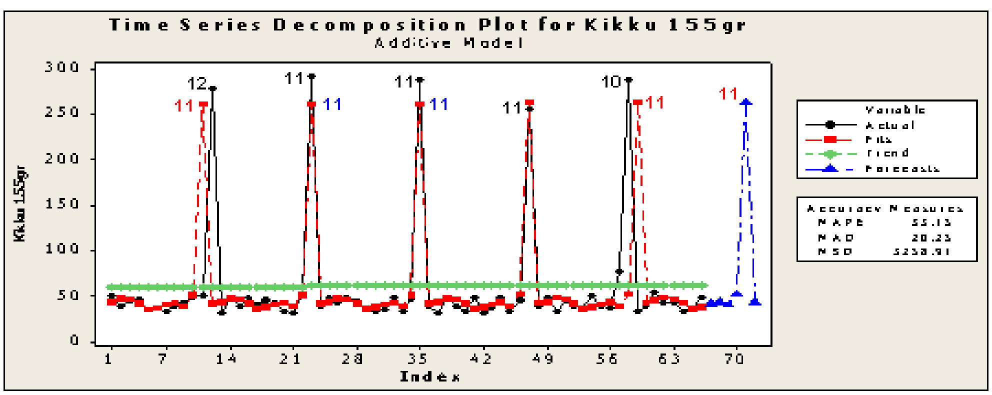


Figure 4. Forecasting results by using the Additive Decomposition Model

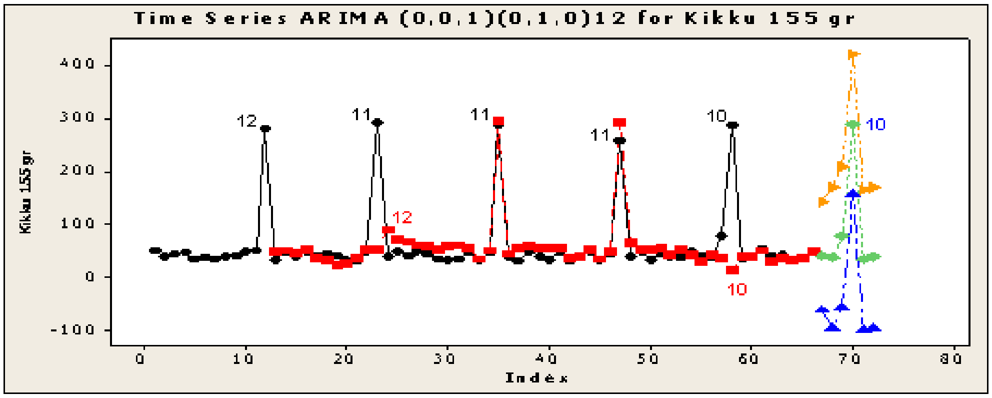


Figure 5. Forecasting results by using the ARIMA(0,01)(0,1,0)^{12} Model

Results of Seasonal ARIMA Model

The results of identification of steps by using time series and ACF (auto-correlation function) plots show that data are not stationer in the seasonality mean. Hence, data should be differentiated seasonaly. After differentiation, ACF pattern tends to cut off after lag 1 and PACF (partial autocorrelation function) dies down. Therefore, the appropriate tentative model is ARIMA(0,0,1)(0,1,0)^{12}.

Parameter estimation and diagnostic check steps show that model is appropriate. The final model is ARIMA(0,0,1)(0,1,0)^{12}, i.e.

$$y_t = y_{t-12} + a_t - 0{,}7960 a_{t-1}. \qquad (8)$$

The graphical forecasting result of this model can be seen in Figure 5. There can be observed clearly that ARIMA model yields nearly the same as Winter's model, i.e. invalid forecast at the second and fifth years when the shift of Eids holiday happens. The forecasts on the next six months at the sixth year show that this model yields appropriate forecasts as Winter's model, i.e. the greatest sales forecast will occur on October and it is correct with the actual condition.

Results of Calendar Variation Model

Based on the results at the identification step, parameter estimation and diagnostic check steps, the calendar variation model can be obtained with that appropriate with data, i.e.

$$y_t = 41{,}14754 + 239{,}05246 CV_t + a_t, \qquad (9)$$

where CV_t is calendar variation variable that has value 1 for month when the Eids holiday happens and 0 for others.

Model in equation (9) shows that the Eids holiday causes the monthly sales Kikku 150 gr sardine significantly increases about 239 thousand cans. This model also explains that monthly sales tend constant around 41 thousand cans at the other months. The result can be seen graphically at Figure 6. Based on this picture, it can be seen clearly that calendar variation model yields valid and reliable forecasts, particularly on the months when calendar variation effects happen. This accuracy of the monthly forecasts happens both in the past and in the coming years.

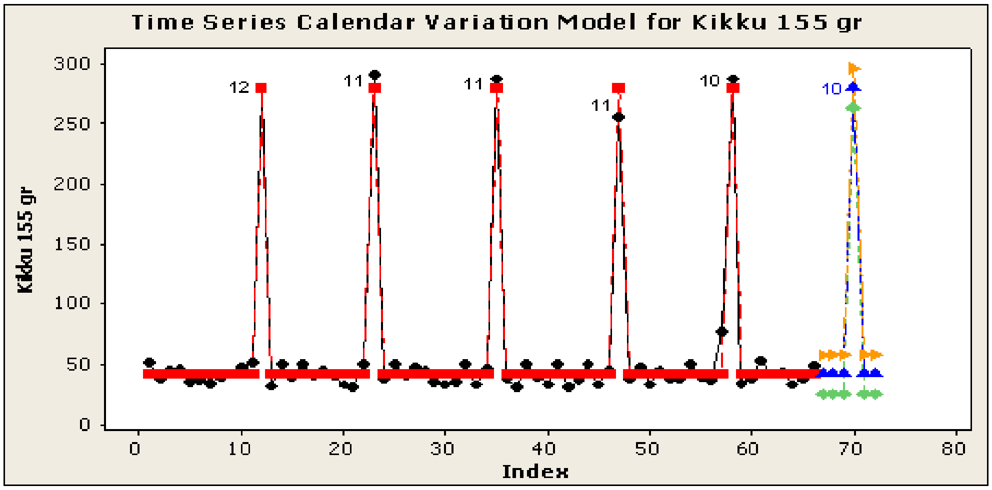


Figure 6. The forecasting result by using Calendar Variation Model

Results of Comparison Study

The focus of this paper is on the comparison of Mean Squares Errors (MSE), Root Mean Squares Errors (RMSE) and Mean Absolute Percentage Error (MAPE) models. The results can be seen in Table 1. This result shows that Calendar Variation model yields the most accurate forecast than any other models. Hence, it can be concluded that Calendar Variation model must or should be used for modeling and forecasting time series data which contain calendar variation effect. This calendar variation caused the other models (Winter's, Decomposition and ARIMA) produce invalid forecasts.

Table 1. The results of forecast comparison among four forecasting models

Errors Forecast Measurement	Time Series Models			
	Winter's Model	Decomposition Model	ARIMA Model	Calendar Variation Model
MSE	4515.81	3258.91	2689.93	68.89
RMSE	67.20 (*)	57.09 (*)	51.87 (*)	8.30 (**)
MAPE	69.68	33.15	29.90	14.34

Note: (*) = errors are not normally distributed
(**) = errors are normally distributed

Additionally, the forecast values on the next six months yielded by these four forecasting models are presented in Table 2. These results show obviously that Calendar Variation model gives the most appropriate forecasts and in line with the actual phenomenon, i.e. the greatest monthly sales happen in October when the Eids holiday occur. The same forecasts are also yielded by ARIMA and Winter's models.

Table 2. The forecasts for the next six months by using four forecasting models

Month (in 2004)	The forecasts on the next six months			
	Winter's Model	Decomposition Model	ARIMA Model	Calendar Variation Model
July	41.420	40.934	40.558	41.148
August	43.591	43.690	36.500	41.148
September	58.535	39.957	76.000	41.148
October	176.092	51.858	287.500	280.200
November	117.822	263.365	33.500	41.148
December	38.253	42.506	37.500	41.148

CONCLUSIONS

Based on the results of the previous sections, it can be concluded that the classical seasonal time series models, i.e. Winter's, Decomposition and ARIMA models, cannot describe the effect of calendar variation and finally yield invalid forecasts. This condition particularly happens in the period when the calendar variation occurs. On the contrary, Calendar Variation model is a model that can depict precisely the impact of the calendar variation effect and finally give valid and reliable forecasts. In general, the results show the importance to do further research on the development of forecasting models that can be applied properly to the time series influenced by people cultures in a certain area.

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