APPLICATION OF THE FUZZY RUEY CHYN TSAUR METHOD TO THE TIME SERIES DATA
(Case Study: Total Exports in East Kalimantan from January 2018 to May 2021)

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ABSTRACT

Markov chains are stochastic processes in which current events depend only on events one step back. The transition probability matrix shows the probability of movement between states in a Markov chain process. The transition likelihood matrix can be used to help forecast future changes. This research has the purpose of forecasting using the fuzzy Ruey Chyn Tsaur method combined with the Markov chain concept. In this study, the determination of interval length was carried out using Sturges and averages based. The results showed that the value of MAPE based on Sturges (5.95%) is lower than the value based on the average (6.02%). In June 2021, forecasting of the total exports was obtained at USD 1,687.17 million for the Sturges method and USD 1,728 million for average based.

Keywords: Average Based, MAPE, Markov Chain, Sturges.

INTRODUCTION

Export is a commercial activity that brings products from within a country abroad, both in the form of finished goods and raw goods (Nurhakim & Satar, 2015). Export activities could encourage a country's economic growth if products made domestically can be widely marketed to various countries (Akbar & Akub, 2022; Hendri & Suwali, 2022; Marheni, 2022; Sinaga et al., 2023; Yuni, 2021). Based on data from (BPS Indonesia, 2021), the total value of Indonesia's exports from January to December 2020 reached US$ 163.31 billion. This condition has decreased compared to the export value from January to December 2019, which had an export value of $167.53 billion. The decline in the total export value in Indonesia in general also occurred in several provinces, including East Kalimantan Province.

Based on the high influence of exports on the economy, it is appropriate for the East Kalimantan Provincial government to strive to increase the quality and quantity of exports by taking the right policies. The interrelationship of past and future conditions is very close to policymaking. In mathematical statistics, this condition is inseparable from the existence of a chain pattern called the Markov chain.

The Markov chain (1906) is a stochastic process in which the current event depends only on previous occurrences(Noh et al., 2015). The Markov chain defines a matrix of transition opportunities that can be acceptable to help forecast future changes. In addition to the Markov chain, time series analysis can also be used for forecasting. Time series analysis is divided into two methods, i.e., classical time series and artificial intelligence. Artificial intelligence is used in computational techniques

(soft computing) that are growing, one of which is the fuzzy time series method which has advantages i.e free asumtion.

The fundamental theory of fuzzy can be combined with time series for forecasting. (Song & Chissom, 1993), as inventors of fuzzy logic, introduced the basic theorems for the first time in 1993. (Tsaur, 2012) continues to develop a combination of fuzzy time series and Markov chain concepts that are applied to time series data to provide more accurate results.

In the fuzzy time series method, the length of the selected interval affects the forecasting results obtained (Sugumonrong et al., 2019; Yenni Safitri, Sri Wahyuningsih, 2018) Therefore, there is freedom for researchers to use the interval length selection method (Rukhansah et al., 2015). There are several ways to determine the length of the fuzzy time series interval. The Struges method and the average-based method are more often used.

Ruey Chyn Tsaur's Fuzzy Time Series method can be applied in economics and trade (Lyani et al., 2020; S, 2016; Tsaur et al., 2021; Yudi, 2018). (Berutu et al., 2011) tried to compare Fuzzy Ruey Chyn Tsaur's method and S. R. Singh's algorithm. The study explained that the results of forecasting using the Ruey Chyn Tsaur algorithm were better than the S. R. Singh algorithm. Furthermore, (Lyani et al., 2020)applied the Fuzzy Ruey Chyn Tsaur method for forecasting, with a mean absolute percentage error (MAPE) value of 0.18%.

Based on the advantages of the fuzzy method that has been introduced by Ruey Chyn Tsaur, researchers are interested in forecasting the total export value in East Kalimantan Province by applying the stochastic-based and average-based fuzzy time series methods.

METHOD

The population of this study is the total exports of East Kalimantan Province, and the sample is the total export value of East Kalimantan Province between January 2018 and May 2021. Purposive sampling techniques are used in the research. Researchers get data on the total export from the official website, http://kaltim.bps.go.id.

Data Analysis Techniques

Descriptive statistics and the fuzzy time series method (Tsaur, 2012), is a data analysis technique used in this study. The following are the stages of the analysis carried out:

1. Make descriptive statistical analysis to find out the general picture of the data through time series plots, minimum data values, and maximum data values.
2. Define the set of universes (U) based on actual data.

$$U = [X_{min} - D_1 \; ; X_{max} + D_2] \tag{1}$$

D_1 and D_2 arbitrary positive number which obtained by trial and error on actual data to get the same U value. X_{min} and X_{max} is minimun and maximum value from the actual data.

3. Arrange the set of universes (U) into several equal-length intervals using the Sturges and average-based methods. The steps for determining Sturges-based intervals are: (a) determining the number of classes (k) as in equations (2); and (b) determining the length of classes (I_S) as in equation (3).

$$k = 1 + 3.33\ \log(N) \tag{2}$$

$$l_s = \frac{X^*_{max} - X^*_{min}}{k} \tag{3}$$

N is the amount of data. While the steps for determining the length of the average-based interval are:

a. Calculates the average value of the absolute difference $(\bar{X})$ between the data X_t and X_{t+1}. To obtain the average value of the absolute difference in each data point, the following formula is used:

$$\bar{X} = \frac{\sum_{t=1}^{N} |X_{t+1} - X_t|}{N - 1} \tag{4}$$

b. Calculates the interval base of the fuzzy set (B) using the average value of the absolute difference of data with the following formula:

$$B = \frac{\bar{X}}{2} \tag{5}$$

c. From the results of calculating the base value above, the base value is adjusted in the following interval base table to determine the range value to be used.

Table 1. Interval base

Range	Basis
0,1 - 1	0.1
1,1 - 10	1
11 – 100	10
101 – 1000	100
1001 - 10000	1000

Source: (Tsaur, 2012)

d. Calculates the length of the interval class (I_B). The length of the interval class can be obtained using the following formula:

$$l_B = (X_{max} + D_2) - \big((X_{min} + D_1)\big) \tag{6}$$

e. Calculates the number of classes (k). The interval basis is used to calculate the number of interval classes that can be obtained using the formula in equation (7).

$$k = \frac{l_B}{B} \tag{7}$$

4. Defines the fuzzy set A_i based on the partition of the set of universes (U) based on Sturges and averages.
5. Fuzzification is based on the degree of membership of fuzzy sets.
6. Form a fuzzy logical relationship (FLR). The formation of FLR starts with determining the current state and next state. If the current state at time $t-1$ is referred to as A_i and moving to the next state at time (t) is referred to as A_j, then the FLR formed is denoted as $A_i \rightarrow A_j$. For example, if the data in January (X_t) has fuzzification A_1., February $X_{(t+1)}$ has fuzzification A_1, March (X_{t+2}) has fuzzification A_2, and April (X_{t+3}) has fuzzification A_5 , then the FLR formed is $A_1 \rightarrow A_1; A_1 \rightarrow A_2$ and $A_2 \rightarrow A_5$.
7. Establish a fuzzy logical relationship group (FLRG) based on a preconceived FLR. The formation of the FLRG on FTS Ruey Chyn Tsaur is slightly different from FTS Song and Chissom. The formation of the FLRG Ruey Chyn Tsaur pays attention to the series in its current state, which follows the basic concept of the Markov chain. For example, after the FLR is formed in the previous step, the FLRG is formed in $A_1 \rightarrow A_1, A_2$, and $A_2 \rightarrow A_5$. In addition, if the formed FLR is attached to an empty set $(A_i \rightarrow \emptyset)$, it is also said to be FLRG.
8. Calculates the transition probability matrix based on the state transition $A_i \rightarrow A_j$.

$$\boldsymbol{R_{ij}^{n}} = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{bmatrix} \tag{8}$$

$$P_{ij} = \frac{M_{ij}}{M_i} \tag{9}$$

9. Perform initial forecasting (F_t) and defuzzification $(\hat{F}_t)$ as per the rules of the FTS Ruey Chyn Tsaur's method.

   Rule 1. If FLRG on A_i transitions to an empty set, $(A_i \rightarrow \emptyset)$ then the initial forecasting result (F_t) is the current state (A_i) of the formed FLRG. While the defuzzification process can be obtained based on the following equation,

$$\hat{F}_t = m_i . P_{ij} \tag{10}$$

Because A_i transitions to an empty set $(A_i \rightarrow \emptyset)$ and the initial forecasting result is the current state itself (A_i) where A_i it only has 1 member, hence the probability, P_{ij} must satisfy the conditions and conditional probability properties of the Markov chain, namely, $P_{ij}^{n} > 0$ and $\sum_{j=1}^{n} P_{ij}^{n} = 1$, where the middle value (m_i) can be calculated based on equation (11).

$$m_i = \frac{(\text{Upper of } U_i + \text{Lower of } U_i)}{2} \tag{11}$$

Rule 2. If the FLRG formed is a one-to-one relation, then the initial forecasting result (F_t) is the next state (A_j) of the formed FLRG. While the defuzzification process can be obtained based on the following equation,

$$\hat{F}_t = m_j . P_{ij} \tag{12}$$

Since the relation is one to one $(A_i \rightarrow A_j)$, the probability of P_{ij} at the time $i = j$ will be 1. While the probability for $i \neq j$ will meet the conditions, the conditional probability properties of the Markov chain are $P_{ij}^n > 0$ and $\sum_{j=1}^{n} P_{ij}^n = 1$.

Rule 3. If the FLRG formed is a one-to-many relation $(A_i \rightarrow A_{j_l}, l = 1,2, \dots, q)$, then the initial forecasting result (F_t) is the next state $\left(A_{j_l}\right)$ of the formed FLRG. While the defuzzification process can be obtained based on the following equation (13),

$$\hat{F}_t = m_{j_1} . P_{ij_1} + m_{j_2} . P_{ij_2 +} \dots + m_{j_{t-1}} P_{ij_{j-1}} + X_{(t-1)} P_{ij_l} + m_{j_{j+1}} P_{ij_{j+1}} + \cdots + m_{j_q} P_{ij_q} \tag{13}$$

when $i = j$, then the transition probability is no longer multiplied by the middle value (m_{j_l}) of the set A_{j_l} but multiplied by the actual data of the state A_i at time (X_{t-1}).

10. Calculates the forecasting trend adjustment value (D_t) based on the equation according to the applicable rules.

Rule 1. If state, A_i communicates with state A_i , starting from state A_i is the current state at time $t - 1$ and transitions to the next state (A_j) at time t where $(i < j)$, then there is an upward transition that causes the adjustment value of D_t, can be determined as follows:

$$D_{t1_{increase}} = (l/2) \tag{14}$$

where l is the length of the interval.

Rule 2. If state A_i communicates with state A_i, starting from state A_i is the current state at time $t - 1$ and transitions to the next state (A_j) at time to t where $(i > j)$, then there is a downward transition movement that causes the adjustment value of D_t, can be determined as follows:

$$D_{t1_{\text{decrease}}} = -(l/2) \tag{15}$$

Rule 3. If state A_i communicates with state A_j, starting from state A_i is the current state at time $t - 1$ and transitions to the next state A_j , where $j = i + s$ at time to t and $(1 \leq s \leq n - 1)$, then there is a jumping up transition displacement that causes an adjustment value of D_t, can be specified as:

$$D_{t2_{\text{increase}}} = (l/2) \times s \tag{16}$$

where s s is the number of leaps forward.

Rule 4. If state, A_i communicates with A_j, starting from state A_i is the current state at time $t-1$ and transitions to the next state A_j, where $j = i - v$ at time to t and $(1 \leq v \leq i)$, then there is a transition shift jumping down which causes an adjustment value of D_t, can be specified as:

$$D_{t2_{(\text{decrease})}} = -(l/2) \times v \tag{17}$$

where v is the number of jumps backwards.

11. Calculates the final forecasting $(\hat{X}_t)$ using the equation (18).

$$\hat{X}_t = \hat{Y}_t \pm D_{t1_{(\text{increase/decrease})}} \pm D_{t2_{(increase/\text{decrease})}} \tag{18}$$

12. Calculate the level of forecasting accuracy using the Mean Absolute Percentage Error (MAPE) method as in equation (19) (Intarapak et al., 2022; Jilani et al., 2019; Liantoni & Agusti, 2020; Pangestu et al., 2018).

$$MAPE = \left(\frac{1}{N}\sum_{t=1}^{N}\frac{|X_t - \hat{X}_t|}{X_t}\right) \times 100\% \tag{19}$$

RESULTS AND DISCUSSION

This research is to apply the Fuzzy Ruey Chyn Tsaur method to the total export data of East Kalimantan for January 2018 – May 2021. The data of the total export value in East Kalimantan can be seen in figure 1. Based on Figure 1, the total export value of East Kalimantan Province fluctuated from January 2018 to May 2021. The highest total export value occurred in May 2021 at USD 1,760.63 million, while the lowest occurred in September 2020 at USD 853.6 million. After being at its lowest point in September 2020, the total export value of East Kalimantan increased from October to December 2020. Throughout January 2018 to May 2021, the highest increase occurred in March 2018 from USD 303.27 million, while a significant decrease occurred in April 2018 to USD 373.83 million.

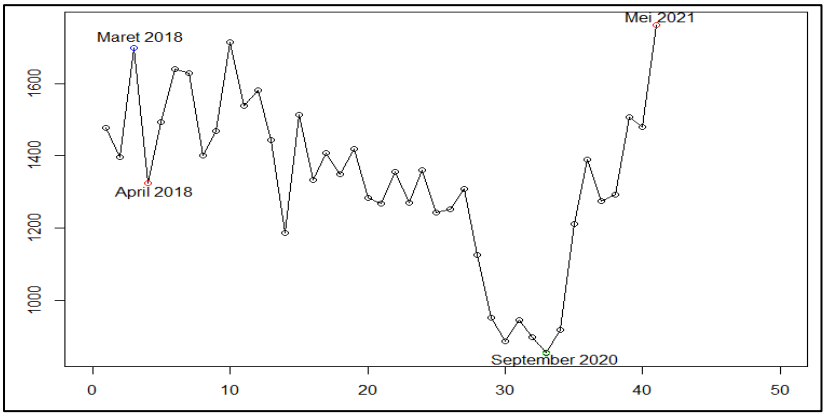


Figure 1. Time series plot of the total export value of East Kalimantan January 2018 – May 2021

Determination of the Set of Universe

Based on the determination of the set of spoken universes, the value of D-min and D-max are required, which is an arbitrary positive number. This study used the values of $D_1 = 0.62$ and $D_2 = 2.37$. D_1 and D_2 obtained by trial and error on actual data to get the same U value, which will be used for determining the length based on the Sturges and average-based class intervals.

$$U = [X_{min} - D_1 ; X_{max} + D_2]$$
$$= [853.62 - 0.62 ; 1{,}760.3 + 2.37]$$
$$= [853 ; 1{,}763]$$

Sturges-Based and Average Based Class Interval Length

The determination of the interval length based on Sturges is obtained from as many as six classes based on equation (2). Furthermore, using the average-based method, we calculate the length of the class interval is obtained from as many as 13 classes, based on (7).

Table 2. Interval length based on Sturges and Average

Sturges		Average	
u_i	m_i	u_i	m_i
$u_1 = [853 ; 1{,}004.67)$	928.83	$u_1 = [853 ; 923)$	888
$u_2 = (1{,}004.67 ; 1{,}156.33)$	1,080.50	$u_2 = (923 ; 993)$	958
$u_3 = (1{,}156.33 ; 1{,}308)$	1,232.17	$u_3 = (993 ; 1{,}063)$	1,028
$u_4 = (1{,}308 ; 1{,}459.67)$	1,383.83	$u_4 = (1{,}063 ; 1{,}133)$	1,098
$u_5 = (1{,}459.67; 1{,}611.33)$	1,535.50	$u_5 = (1{,}133 ; 1{,}203)$	1,168
$u_6 = (1{,}611.33 ; 1{,}763]$	1,687.17	$u_6 = (1{,}203 ; 1{,}273)$	1,238
		$u_7 = (1{,}273 ; 1{,}343)$	1,308
		$u_8 = (1{,}343 ; 1{,}413)$	1,378
		$u_9 = (1{,}413 ; 1{,}483)$	1,448
		$u_{10} = (1{,}483 ; 1{,}553)$	1,518
		$u_{11} = (1{,}553 ; 1{,}623)$	1,588
		$u_{12} = (1{,}623 ; 1{,}693)$	1,658
		$u_{13} = (1{,}693 ; 1{,}763]$	1,728

Fuzzification of Actual Data

The fuzzification process for the total export value of East Kalimantan Province from January 2018 to May 2021, as Table 3.

Table 3. Fuzzification using Sturges and average

Year	Month	X_t	*Fuzzification**	*Fuzzification***
2018	January	1,476.8	A_5	A_9
	February	1,395.25	A_4	A_8
	⋮	⋮	⋮	⋮
2021	April	1,478.76	A_5	A_9
	May	1,760.63	A_6	A_{13}

Fuzzification based on Sturges*; Fuzzification based on average**

Referring to Table 3, the total export in January 2018 is 1,476.80. On the Sturges method, present the interval (1,459.67; 1,611.33) which is the 5th fuzzy set (u_5), so the fuzzification is A_5. In addition, the number of total exports based on the average-based method in January 2018, the fuzzification is A_9, because the interval is (1,413; 1,483) belonging to the 9th fuzzy set A_9.

Fuzzy Logical Relationship (FLR)

The determination of FLR is based on the current and the next state. For example, January 2018 is the current state (X_t) with a fuzzification value of A-5, and February 2018 is the next state (X_{t+1}) with a fuzzification value of A_4. The FLR formed between January 2018 and February 2018 is $A_5 \rightarrow A_4$. The complete FLR for each month from January 2018 to May 2021 based on Sturges and averages can be seen in Table 4.

Table 4. FLR based on Sturges and average

Months	FLR*	FLR**
Jan 2018 – Feb 2018	$A_5 \rightarrow A_4$	$A_9 \rightarrow A_8$
Feb 2018 – Mar 2018	$A_4 \rightarrow A_6$	$A_8 \rightarrow A_{13}$
⋮	⋮	⋮
Mar 2021 – Apr 2021	$A_5 \rightarrow A_5$	$A_{10} \rightarrow A_9$
Apr 2021 – May 2021	$A_5 \rightarrow A_6$	$A_9 \rightarrow A_{13}$

FLR based on Sturges*; FLR based on average**

Fuzzy Logical Relationship Groups (FLRG)

The FLRG in Ruey Chyn Tsaur fuzzy time series pays attention to the series in its current state. It follows the basic concept of the Markov chain, which depends only on events one step go back (X_t) and does not depend on previous events $(X_{t-1}, X_{t-2}, \dots, X_{t-n})$. FLRG is not only for one-to-many relations; FLR that has only one next state and FLR that goes to empty sets is also said to be FLRG. For example, FLR which only has one next state, in January 2018–February 2018 is $A_5 \rightarrow A_4$, then FLR in February 2018–March 2018 is $A_4 \rightarrow A_6$, then FLRG is formed consecutively $A_5 \rightarrow A_4$ and $A_4 \rightarrow A_6$. The FLR which has a one-to-many relationship, is that FLR in November 2018–December 2018 is $A_5 \rightarrow A_5$, and FLR in December 2018–January 2019 is $A_5 \rightarrow A_4$, then the FLRG formed is $A_5 \rightarrow A_5, A_4$. Another example of FLR that has a one-to-many relationship is FLR in March 2021–April 2021, which $A_5 \rightarrow A_5$, and FLR in April 2021–May 2021, $A_5 \rightarrow A_6$, then the FLRG formed is $A_5 \rightarrow A_5, A_6$. In both examples, FLR has a one-to-many relationship with the same current state.

However, due to the different time differences as in the example above, November 2018–December 2018, December 2018–January 2019, and March 2021–April 2021, April 2021–May 2021, the two examples cannot be grouped into the same group. The complete FLRG can be seen in Table 5.

Table 5. FLRG based on Sturges and average

Group	FLRG*	FLRG**	Group	FLRG*	FLRG**
1	$A_5 \rightarrow A_4$	$A_9 \rightarrow A_8$	21	$A_1 \rightarrow 5A_1, A_3$	$A_6 \rightarrow A_8$
2	$A_4 \rightarrow A_6$	$A_8 \rightarrow A_{13}$	⋮	⋮	⋮
⋮	⋮	⋮	25	$A_5 \rightarrow A_5, A_6$	$A_4 \rightarrow A_2$
⋮	⋮	⋮	⋮	⋮	⋮
20	$A_2 \rightarrow A_1$	$A_8 \rightarrow A_6$	34	–	$A_9 \rightarrow A_{13}$

Transition Probability Matrix

The determination of the transition probability matrix based on Sturges, and averages is obtained based on the FLRG formed in Table 5. This is used to obtain the transition probability value in a matrix, which is 6×6 for the Sturges method ($\mathbf{R}^*$) and 13×13 for average based ($\mathbf{R}^{**}$). Each element in this matrix is a probability value obtained from equation (8). For example, $P_{A_1A_1}$ is the transition probability of the state A_1 to A_1, $M_{A_1A_1}$ is the number of repetitions from state A_1 to A_1,and M_{A_1} is the number of members included in the next state (A_j) which obtained from the 21st group in Table 5. $P_{A_1A_1} = \frac{M_{A_1A_1}}{M_{A_1}} = \frac{5}{6}$.

$$\mathbf{R}^* = \begin{bmatrix} 5/6 & 0 & 1/6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/7 & 4/7 & 1/7 & 1/7 & 0 \\ 0 & 0 & 1/4 & 3/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 2/4 & 1/4 \\ 0 & 0 & 0 & 1/2 & 0 & 1/2 \end{bmatrix};$$

$$\mathbf{R}^{**} = \begin{bmatrix} 2/3 & 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 0 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Initial Forecasting and Defuzzification

Initial forecasting and defuzzification are obtained based on the rules contained in equations (10), (12), and (13). For example, in February 2018, which is included in the 1st group, $A_5 \rightarrow$

A_4, based on rule 2, where if the FLRG formed is a one-to-one relationship, the initial forecasting result (F_t) is the next state itself, A_4. Furthermore, the defuzzification value can be calculated using equation (12), and the value of the transition probability from A_4 is a value of 1, because the terms and properties of the Markov chain are $P_{ij}^n > 0$ and $\sum_{j=1}^{n} P_{ij}^n = 1$, so that the result of defuzzification is obtained at 1,383.83. Another example is in May 2019, which is included in the 14th group, $A_4 \rightarrow 3A_4, A_3$, based on rule 3, if the FLRG formed is a one-to-many relation, then the initial forecasting result is the next state itself, which is $3A_4, A_3$ (Table 6).

Table 6. Defuzzification based on Sturges and average

Year	Month	X_t	F_t^*	$\hat{F}_t^*$	F_t^{**}	$\hat{F}_t^{**}$
2018	January	1,476.8	-	-	-	-
	February	1,395.25	A_4	1,383.83	A_8	1,378.00
	⋮	⋮	⋮	⋮	⋮	⋮
2021	April	1,478.76	A_5, A_6	1,596.38	A_9	1,448.00
	May	1,760.63	A_5, A_6	1,582.96	A_{13}	1,728.00

Final Forecasting

The tendency to forecast values in Ruey Chyn Tsaur fuzzy time series method aims to reduce of forecasting errors. The trend of forecasting values applies at the time of the one-to-many relation, and each transition moves more than two of the states, A_i towards A_j. For example, in March 2018, based on Table 5 of the FLRG formed, it was $A_4 \rightarrow A_6$. The calculation for the trend setting value uses rule 3 with equation (16), where s is the number of forward jumps from the state $A_4 \rightarrow A_6$, i.e., 2. Another example is the computation of the trend observed on average for August 2018. The FLRG is $A_{12} \rightarrow A_{12}, A_8$, uses rules 2 and 4 with equations (15) and (17).

Table 7. Tren tendency based on Sturges and averages

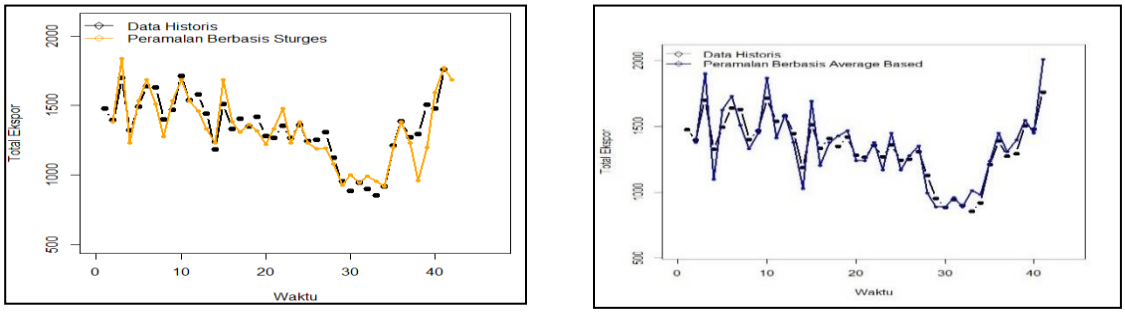
Year	Month	X_t	D_t^*	D_t^{**}
2018	January	1,476.8	-	-
	February	1,395.25	0	0
	⋮	⋮	⋮	⋮
2021	April	1,478.76	0	0
	May	1,760.63	151.67	280

After calculating the trend tendency for forecasting, the final forecast can be seen in Table 8.

Table 8. Final forecasting

Year	Month	X_t	$\hat{X}_t^*$	$\hat{X}_t^{**}$
2018	Januari	1,476.8	-	-
	February	1,395.25	1,383.83	1,378
⋮	⋮	⋮	⋮	⋮
2021	April	1,478.76	1,596.38	1,448
	Mei	1,760.63	1,734.63	2,008

Based on Table 8, it is getting the Figure 2 to compare actual data and forecasting.



(a) (b)

Figure 2. Forecasting results Based on Sturges 2(a) and average 2(b)

Table 9. MAPE based on Sturges and average

Method	MAPE
Sturges	**5,86%**
Average Based	6,02%

To calculate the forecasting for the next period, June 2021, the current state (May 2021) is in state A-6 (Table 2), and the next state is an empty set (∅). The FLR and FLRG of May 2021 and June 2021 are $A_{13} \rightarrow \emptyset$. According to rule 1, if the formed FLRG transitions to an empty set, the initial forecasting result (F_t^*) is A_6. The defuzzification uses equation (10), and the transition probability is 1. So that, the forecasting value of one period in the future is obtained as in Table 10.

Table 10. Forecasting in June 2021

Period	$\hat{F}_{42}^{*}$	$\hat{F}_{42}^{**}$
June 2021	$\hat{F}_{42}^{*} = m_6 . P_{ij}$ $= 1{,}687.17(1)$ $= 1{,}687.17$	$\hat{F}_{42}^{**} = m_{13} . P_{ij}$ $= 1{,}728(1)$ $= 1{,}728$

Forecasting based on Sturges*; Forecasting based on average**

CONCLUSION

Based on the results and discussions, the conclusions of this study are the value of MAPE based on Sturges (5.95%) is lower than the value based on the average (6.02%), and in June 2021, forecasting of the total exports was obtained at USD 1,687.17 million for the Sturges method and USD 1,728 million for average based.

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